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TOWARDS A GENERAL FORECASTING MODEL FOR CRISIS MONITORING: PREDICTING EVENTS IN CHINA AS TEST CASE

Richard P. Y. Li

Sponsored by

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This report summarizes the development and testing of the CEWMFC interactive forecasting system for monitoring political crises and policy changes in China. The first chapter presents an overview of the project, discusses its objective and its major accomplishments and delineates the structure of the report. The second chapter discusses the philosophy and the design of the CEWMFC System, and describes the forecasting model and the modeling strategy the system employs. The third chapter delineates the structure

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and function of the CEWMFC system. The fourth chapter summarizes the results of sensitivity analysis and forecast reliability tests performed on the CEWMFC system, and defines the hypotheses and the indicators employed in data analyses. Appendix A then presents a technical description of the mathematical foundation of the CEWMFC system. Appendix B describes the coding procedure for constructing the Chinese crisis and policy indicators. Appendix C describes the statistical models used in the data analyses.

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TOWARDS A GENERAL FORECASTING MODEL FOR CRISIS MONITORING: PREDICTING EVENTS IN CHINA AS A TEST CASE

by

Richard P.Y. Li
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Introduction

This report summarizes the results derived from a project to develop an interactive computer forecasting system for crisis early warning and monitoring. The project was conducted at Michigan State University under the sponsorship of the Defense Advanced Research Projects Agency's Cybernetics and Technology Office as a part of its Crisis Management Program.

1.0 Project Overview

The project was conceived in response to Crisis Management Program's need for auxilliary computer software to augment the forecasting capability of its existing Early Warning and Monitoring System (EWAMS). This auxilliary system, designated the Crisis Early Warning and Monitoring Forecaster (CEWMFC), generates through leading indicators multivariate forecasts of crisis frequencies and probabilities as distinct from the subjective forecasts of the existing system.

The EWAMS generates, from an inventory of crisis indicators, estimates of the probability that the observed value at a given time point reflects an abnormal deviation from "usualness", which is taken to indicate a crisis occurence. Since these probabilities are estimated for the existing observations, the analyst must intuit the likelihood of future crisis events.

Supplementing the foregoing subjective forecasting capability, the CEWMFC system offers a frequency forecasting capability through which one could numerically extrapolate future crisis probabilities from past trends and through coterminous leading indicators. The use of mathematical routines to project future values from past trends minimizes the biases attendant to subjective evaluation and estimation of future likelihood based on current values.

The CEWMFC system is also fitted with a database and a database manager. The database is comprised of various measures of Chinese domestic political crisis and policy indicators. Tests of the system's forecast reliability are performed with these indicators and the results have verified several hypotheses regarding Chinese politics.

1.1 Research Objectives

In order to enhance the forecasting capability of the EWAMS system this study has undertaken the following objectives:

1. To develop an interactive computer forecasting system which, with minimum user inputs, will automatically generate in real time crisis predictions from a single series of observations or through multiple leading indicators.
2. To construct various indicators for the purpose of testing the forecast reliability of the CEWM system using hypotheses linking elite crisis behavior to policy changes in China.

Attaining the above objectives has enhanced the capabilities of the EWAMS system in terms of

- its computer-based analytic and forecasting capabilities
- its use of advanced forecasting methodology
- its development of crisis indicators

1.2 Major Research Accomplishments

1. Interactive, automated real time computer forecasting system

   A computer forecasting software system has been developed that permits direct user-system interaction, performs automated model fitting, and generates instantaneous forecasts. The system also permits the combining and the updating of forecasts. With a minimum

of user input the system is largely self-driven.

2. Indicator Construction

Various crisis and domestic policy indicators have been constructed. These indicators are developed from elite biographical data sources and aggregated from economic and defense data.

3. Testing of Hypotheses

Use of the indicators to test the system's forecasting reliability has permitted the verification of two alternative hypotheses pertaining to factional fractionation in Chinese politics during crisis and to the relationship between crisis and policy changes.

1.3 Structure of the Report

This report is divided into three parts. Part I gives a non-technical general description of the project. It covers

- modeling philosophy, relating to design, mathematical background, and system testing,
- computer programming in relation to developing the various functional and structural attributes of the system,
- results of the system's forecasting reliability tests linking the crisis and the policy indicators.

Part II provides technical summaries of the mathematical background, the coding procedure, and the statistical models used in designing and testing the CEWMFC system.

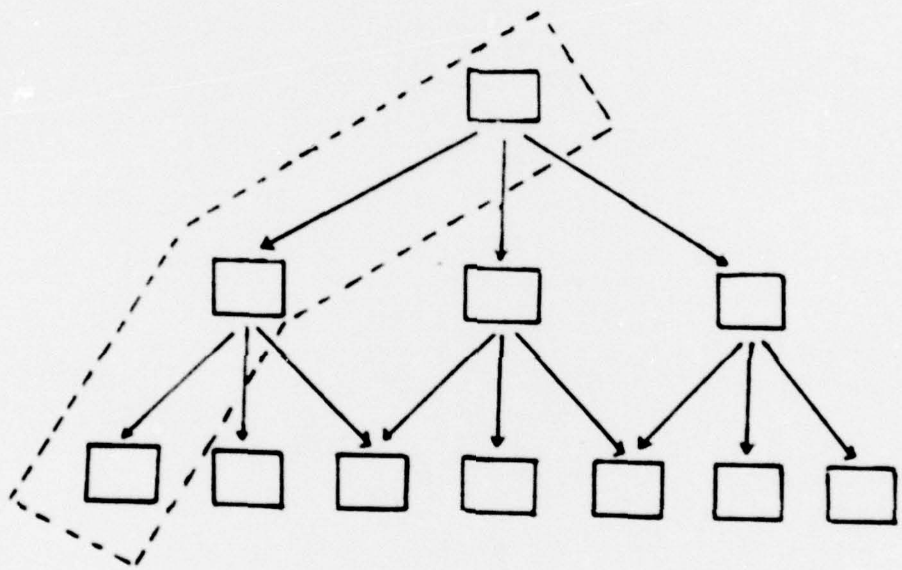
2.0 General System Philosophy

CEWMFC is an integrated data management and forecasting system. It links a database to a forecaster in an integrated unit that provides for the permanent storage and periodic updating of crisis data for which forecasts are generated in real time. For the purpose of early warning and monitoring of crises under dynamic conditions, the system is designed to operate in real time permitting direct user-system interaction. This interface is facilitated by a system of interactive hardware units and software packages. Through this conduit of hardware and software the user performs a range of statistical and manipulative tasks resulting in short-range multivariate forecasts of crisis frequencies and crisis probabilities.

Moreover, CEWMFC is designed to operate as an automated system that requires minimum user operational guidance. All technical modeling decisions are pre-programmed into driver routines directing model fitting and forecasting. This feature of the system simplifies the user-system interface and renders it accessible to users with limited knowledge of modeling and forecasting methodology. The system so created is thus simple to use, efficient to operate, and accessible to technical and non-technical users alike.

Nonetheless, certain system maintenance must still be performed. The inputing, editing, and updating of data demand some familiarity with programming general information systems. It is therefore assumed that the system will be supported by a technical staff that maintains the integrity of the database and the operational efficiency of the database manager and of the forecaster.

FIGURE 1
OVERLAY PROGRAM STRUCTURE



2.1 System Design Operational Mode

The system maintains an array of software packages encompassing over 100 subroutines, supported by ARPA's Demonstration and Development Facilities (DDF) hardware units. The software packages are arranged in a hierarchical structure of layers of controlling main programs and computational subroutines as Figure 1 illustrates. At the top of this structure is the interface component that elicits, receives, and forwards user commands, and that arranges and activates various major components of the system. Immediately beneath this level are the software packages subdivided into one of two general categories: those that manage the database and those that direct the modeling and the forecasting tasks of the system. These include various routines and subroutines that regulate the filing, retrieving, securing, and editing of data and execute the model fitting and forecasting.

Program execution is expedited in terms of a branching process commonly known as "overlay structure". This process permits the system to focus on a sequence of tasks performed by a chain of routines that constitute merely a branch in the overall structure such as that enclosed by dotted lines in Figure 1. By focusing on a branch of the system to complete a particular sequence of tasks, this branching process ensures significant saving in space allocation which is essential for the efficient use of a minicomputer such as the PDP-11/70 of the EWAMS system.

2.2 The Model

The modeling and forecasting softwares are built upon a time series methodology. A class of discrete-time models known as the "autoregressive-moving average vector" (ARMAV) models form the mathematical fouhdation of the CEWMFC system. As section A.1 illustrates, these models are systems

of extended "autoregressive-moving average" equations describing a structure of dynamic interrelationships:

$$\underset{\sim}{Y}_t = \underset{\sim}{\phi}_1 \underset{\sim}{B}^d \underset{\sim}{Y}_{t-1} + \cdots + \underset{\sim}{\phi}_m \underset{\sim}{B}^d \underset{\sim}{Y}_{t-m} + \underset{\sim}{a}_t - \underset{\sim}{\theta}_1 \underset{\sim}{B}^d \underset{\sim}{a}_{t-1} - \cdots - \underset{\sim}{\theta}_n \underset{\sim}{B}^d \underset{\sim}{a}_{t-1}$$

where the $\underset{\sim}{Y}_t$'s are px1 vectors of lagged time series, and $\underset{\sim}{a}_t$'s are px1 vectors of white noise series, the $\underset{\sim}{\phi}$'s and $\underset{\sim}{\theta}$'s are pxp matrices of auto-regressive and moving average parameters respectively, and the $\underset{\sim}{B}^d$'s are matrices of difference operators. These elements are further discussed in section A.1.

The model assumes the dependence of each series, Y_{it}, on its own pasts, Y_{it-j}, for $j>0$, and on the past noises, a_{it-j}, for $j \geq 0$, as well as the dynamic interdependence between series, e.g., Y_{1t}'s dependent on Y_{pt-j}. In other words, the model links multiple input and output series via a network of causal feedback relationships. The dependence of successive states within series represents the memory dynamics that filter through time, whereas the interdependence between series represents their dynamic feedback relationships.

While memory dynamics from the past are assumed to filter through time to the present no reverse process is assumed. For instance, the model does not permit dependence of current noise, a_{it}, on past Y_{it-j}, nor does it permit interdependence between successive noise series.

2.3 Modeling Strategy

The modeling strategy for the CEWMFC system merges a time series modeling procedure with a step-wise regression approach. A popular time series modeling procedure attributed to Box and Jenkins (1970) prescribes a three-step approach, encompassing identification, estimation, and diagnostic checking. Identification is the process by which a tentative model is entertained. Through estimation we derive the parameter estimates. Through diagnostic checking we test for model inadequacy through analysis of residuals. In contrast, the step-wise regression

approach calls for successive approximations of higher order models. We thus identify time series models via successive approximations of model to data, we estimate models by nonlinear least-squares method, and we check the final approximating model by a goodness of fit test.

The merging of step-wise regression approach to time series analysis is essential to the development of an automated modeling procedure, as the normal time series identification method involves rather technical procedures. Thus, as in step-wise regression the present approach seeks to explain away as much of the variations in time series as possible via successive model approximations. Because time series dynamics are caused by temporal dependence between successive observations, temporal dependence can easily be successively approximated by higher order models. Since it merely entails successively incrementing the model by an order at a time, the procedure is mechanical and easily programmable.

2.3.1 Identification

Using a recursive search procedure we can design a computer program that automatically scans through ascending orders of time series models for an optimal model. It starts with the basic first order autoregressive vector process and steadily progresses through higher orders of autoregressive-moving average vector processes. As the search proceeds through higher order autoregressive-moving average vector models by increments of one in the autoregressive and the moving average components, a pair of models is compared at each increment to determine whether the added parameters in the higher order model enhance its explanatory power. This relative explanatory power of the models is ascertained by comparing their residuals for significant differences. This search procedure continues until no difference is observed and the lower order model is retained as the optimal case.

Starting with ARV(1) the model is incremented to ARMAV(2,1), and then to ARMAV(3,2) and so on. Each increment adds an order to the autoregressive and the moving average components. These are known as the "mainline" models, because they represent linear extensions from the basic ARV(1) form. There are alternative combinations of autoregressive and moving average components to each mainline model but they will not be considered until the best fitting mainline model is chosen. For instance, as Figure 3 in section A.2.1 shows, alternative combinations to the mainline ARMAV(3,2) model include ARMAV(3,3), ARMAV(4,2), ARMAV(3,1), ARMAV(2,2). After these models are compared and the best model chosen, a search for superfluous parameters in the final model concludes the search procedure. Superfluous parameters are identified by comparing models in descending orders of moving average parameters, such as ARMAV(4,2), ARMAV(4,1), and ARMAV(4,0).

Since most data are not free of sampling error, a test of significance is used in model comparison. The test compares each pair of models to determine whether the observed improvement in goodness of fit between models is "statistically significant" or is attributable to random perturbations. The test compares the variance-covariance of the residuals for pairs of models. The test statistic has an F distribution. As it is shown in section A.2.2.4, the residual variance-covariance for our model is best described in matrix form (as our model consists of a system of equations which are interdependent and autocorrelated). The F test compares the determinants of the residual variance-covariance matrices for pairs of models. By setting up a critical test value at a given significant level under the null hypothesis that there is no significant difference in the models' explanatory powers, we can build into our program a decision rule to compare our F test

statistic and to accept or reject a higher order model as a better alternative to the lower order case. This search for a better higher order model continues until the test statistic reveals a value that, when compared with the critical value, suggests acceptance of the null hypothesis and retention of the lower order model. When no further improvement is possible the program will automatically determinate all further computation.

For a fixed critical value it is possible to arrive at several significant models, all of which pass the critical level test. But the models may not provide identical forecasting results. Thus, an additional test of forecast reliability is also performed for comparison among the alternative significant models. This reliability test compares the "mean square error" of the forecasts (sum of squares of the differences between the forecasts and actual observations of alternative significant models. The best model is one with the minimum forecast mean square error. Accordingly, the model so chosen not only is statistically significant but also provides the best forecasts.

Finally, having chosen the most powerful forecast model, we go through a final check for superfluous moving average parameters. According to regression logic, most time series dynamics can be captured by the systematic (the autoregressive in the present case) part of a model, and only noise variations should exist in the error structure. Thus, it is entirely possible to represent a time series by purely autoregressive models or by coupling models with high orders of autoregressive parameters to low orders of moving-average parameters. By concurrently incrementing the autoregressive and the moving average components, it is possible to lead to overfitting later. To test this possibility, a final check for superfluous moving average parameters in the selected forecast model is performed. The model is successively examined for decreasing orders of moving average parameters. All zero moving

average parameters are deleted from the model before forecasting is attempted.

2.3.2 Estimation

A nonlinear least-squares method is used in parameter estimation. Since the nonlinear least-squares method uses an iterative search procedure to arrive at the final maximum likelihood estimates, it requires the user to supply the initial values to start off the iterative search procedure. How these initial values are determined affects the quality of the final estimates. As it is beyond the means of most general users to supply good initial values, we employ a three-step estimation procedure that automatically derives the initial values which are ordinarily supplied by the user. The approach develops a method whereby the initial parameter estimates for initializing nonlinear least-squares estimation is automatically derived within the system. The steps are as follows.

(1) A linear autoregressive vector (ARV(k)) model is first fitted to data by means of the linear least-squares method. Estimation is direct requiring no user inputs.

(2) The linear parameters estimated are then transformed, by an inverse function, into a corresponding set of nonlinear autoregressive-moving average vector (ARMAV(m,n)) parameters. The function links the linear parameters of infinite order to linear combinations of nonlinear parameters, as (A.3.2) suggests. Equating the linear least-squares estimates to the inverse function parameters yields the necessary information for computing the corresponding nonlinear autoregressive-moving average estimates. Since the inverse function parameters are linear combinations of the autoregressive-moving average parameters, it is possible to compute the latter through a recursive procedure once the former are known.

(3) The transformed autoregressive-moving average estimates are then taken as the initial values for starting off the iterations in nonlinear least-squares estimation. The nonlinear least-squares estimation method uses the Fletcher-Powell (Fletcher and Powell, 1963) minimization method to minimize the residual sum-of-squares matrix of the vector model. More detailed technical discussion is given in section A.2.4.

2.3.3 Diagnostic Checking

A check for goodness of fit of the selected forecast model concludes the modeling procedure. This final stage further tests the adequacy of the model. It involves a test for systematic trends in the residuals of the fitted model. The presence of such trends indicates model inadequacy. Several statistics are employed at this stage. They include the residual autocorrelations and partial autocorrelations, the cross-correlations between the residuals and the time series, and the associated Q and S significance tests. These statistics address not only trends within series but also trends between series.

2.4 Forecasting

The objective of time series modeling is the development of optimal forecasts. These forecasts produce the least amount of forecast mean square error in relation to all possible alternative forecasts.

These minimum mean square error forecasts are recursively computed from the forecast equations according to some rules of conditional expectation. The results contain both elements of naive autoregressive-moving average forecasts and leading indicator forecasts. The CEWMFC system thus can generate both naive forecasts from a single series or forecasts by leading indicators with several series.

3.0 The Computer Forecasting System

The CEWMFC system is programmed in ANSI standard Fortran IV language. The system operates in real time to permit direct system-user interface. To simplify system operation, all necessary user command inputs are prompted by system solicitations, which appear in the form of questions, such as "number of forecasts?" This streamlines user input commands in relation to a prearranged sequence of operations. Moreover, the user is also aided by "help" routines which provide additional explanations of and instructions for the actions required. As most technical decisions are internalized in the driver routines, the solicitations are limited to simple, nontechnical decisions.

3.1 System Functional Attributes

As a management aid for crisis monitoring, the CEWMFC system generates crisis frequencies and crisis probabilities. Its forecasting capability is enhanced by the following system attributes.

3.1.1 Interactive and Real Time

The system is designed to interface directly with the user via graphics terminals. User commands induce immediate responses from the system. This is particularly important for solving sequential decision problems that frequently confront the crisis analyst. As crisis events unfold in rapid succession and as new data become available with each unfolding event, it is imperative that forecasts be generated quickly in order to ensure continuous monitoring of events. Thus, the ability of the system to interact in real time with the user is an essential feature of the CEWMFC system. The automated modeling capability also enhances the system and its forecast

updating capability which further minimizes user decision time and ensures rapid fine tuning of previous forecasts.

3.1.2 Automated Modeling

While most existing time series techniques (e.g. the Box-Jenkin's (1970) approach or Brown's (1966) exponential smoothing) require the user to identify the correct time series model, the CEWMFC system uses an automated procedure to identify the optimal forecasting model. Through a combination of significance test and forecast mean square error test nested in a recursive model search routine, the CEWMFC system automatically identifies and estimates the optimal model. Such technical decisions normally required in model building are incorporated in driver routines. Thus, relieved of the tasks of making technical decisions, most non-technically trained users may find the system simple to operate.

In addition, some optional statistical maneuvers are available for the trained specialists. These statistics permit the user to manipulate complex time series (e.g., nonstationary series) using various transformational procedures and to check for the adequacy of the optimal model determined by the system.

3.1.3 Multiple Forecasting Capabilities

CEWMFC system maintains multiple forecasting capabilities. The system generates both "projective" and "objective" forecasts (Andriole and Young, 1977). Projective forecasts are trends extrapolated from single series of observations. Objective forecasts are forecasts generated by leading indicators. These forecasts may be used interchangeably to address a variety of research needs. Besides providing naive numerical projection, they may be used to study interaction patterns, systems and stability analyses, and optimal control analysis. Some of these potential applications are discussed

in a series of monographs by Zinnes, Gillespie and their co-workers (1978a, 1978b, 1979).

The system updates forecasts with each new piece of information using a simple "adaptive forecasting" method which does not require refitting the entire forecasting function. The method adjusts for the forecast error, revealed by the latest piece of information entering the system, through a simple weighting function (Section A.3.4). The result is a continuous series of adjustments that fine tunes the forecasts over time. Such adjustments are localized about single pieces of information; they are not sensitive to abrupt changes or to steady long-term drifts in the time series. Updating must therefore be limited to short-range forecasting.

The system maintains combining forecasts capability. Forecasts generated by several alternative methods are combined when they are available for a given indicator. The combined forecasts have been shown to yield more accurate results than their original values (Dickinson, 1975). The method incorporated in the CEWMFC system combines several forecasts, each weighted by its estimated sum-of-squares of past forecast errors. The forecasts to be combined are not restricted to those generated within the system. Forecasts generated externally may also be incorporated into the system. The system thus serves an integrative function in linking forecasts from alternative forecasting systems.

For each frequency forecast the system computes its associated probability. The probability is derived from a normal distribution and it measures the likelihood of the forecast to deviate from an average of past values by its observed magnitude if only random effects were present. Abnormal deviations imply crises.

3.1.4 File Manipulation Capability

The CEWMFC system maintains a database manager which ensures the accessibility, integrity and security of data entered into the system and stores the data generated within the system. The system receives incoming data from external sources; it creates new data using various transformations; it edits existing data in accordance with user commands.

The system maintains four separate file types, including the directory file, the descriptor files, the data files, and the model files. These files identify the countries for which data are stored, store the data, describe the content and arrangement of the data stored, and store the statistical results of model fitting and forecasting.

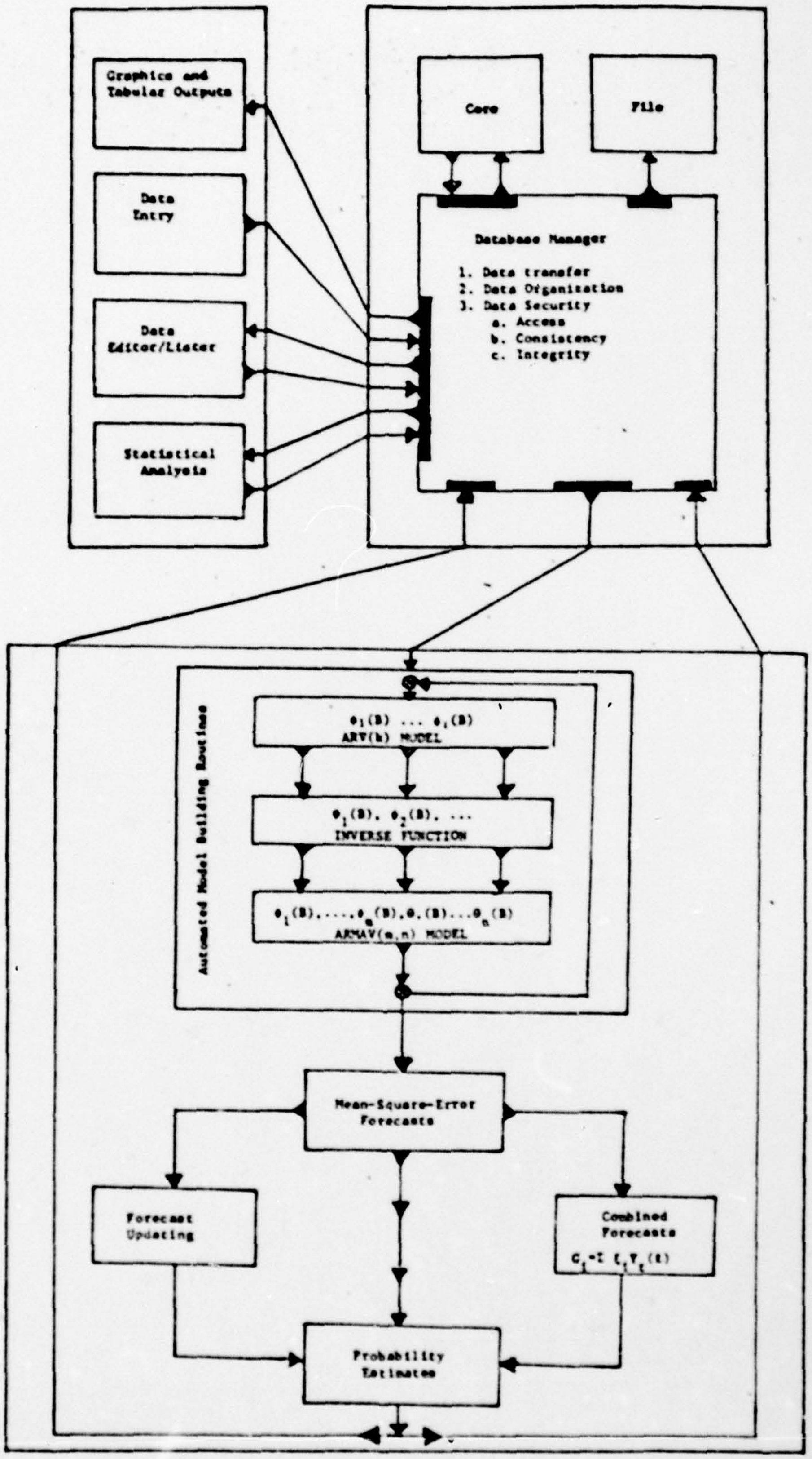
Moreover, the system is capable of storing and manipulating data for a maximum of 100 countries and for each a total of 100 indicators. For each indicator the system may maintain a maximum of 500 observations. Finally, the system has the capability to accomodate data for a total of five estimated models.

Most types of files, excepting the data files, are programmed in unformatted direct access (random) form that permits direct user access to any part of the file structure, eliminating the need to search through a prearranged sequence of information as in formatted sequential files. Such a flexible format structure shortens operational time required in locating data within the system. Nonetheless, no direct access to the files independent of the system is available to the user. Since the system requires a sequence of executions to effect data editing and data deletion, it does secure the data against unintended alteration or destruction.

3.2 System Structural Attributes

The CEWMFC system is comprised of three main structural components ,

Figure 2
CEWMFC System Structure



as Figure 2 illustrates. They are

a. A set of command modules which monitor the inputing, the outputing, and the editing of data; the modules regulate the graphic displays of the forecasts; and control the implementation of statistical analysis to identify, estimate, and generate forecasts forecast functions. These modules are tied together by a main program which is the interface that facilitates the real time interaction between the system and the user.

b. A database manager which regulates the filing, retrieval, securing, editing, and storage of data.

c. A system of statistical routines that identify, estimate, and generate forecasts from the forecast functions as well as perform various auxilliary statistical computations.

3.2.1 The Control Modules

The control Modules consist of four components tied together by a main program that serves as the interface between the system and the user. This interfacing routine receives and deciphers incoming user commands and activates various components of the system. Subsumed under this main program are four control modules:

(1) A data entry control module that regulates the inputing and the outputing of data. This module controls the creation of data and its entry into the system. Data creation is achieved through one of the ensuing transformations:

-logarithmic

-multiplicative, indicator by scalar

-multiplicative, indicator by indicator

-exponential power

-linear combination

-regular difference

-seasonal difference

(2) A data editor/lister that lists data and system information (e.g., name of country and its associated descriptor file) and edits such information as the user specifies.

(3) A data display control module that regulates the graphical and tabular outputs from the system.

(4) An analytical control module that coordinates the executions of an integrated system of time series model fitting and forecasting algorithms.

3.2.2 Database Manager

The database manager includes a set of programs that maintains the security and organizes the storage of data. The data include raw and transformed observations, the forecasts and their associated statistics, and the parameter estimates. This manager monitors user access to the data file, maintains the internal consistency of data, secures the data against inadvertant alternation or destruction, and organizes the filing and storage of data. The databse itself is subdivided into a core and a file structure.

3.2.3 Analytic Routines

The analytic routines form an integrated system of algorithms which identify, estimate, and check the goodness of fit of the forecast functions, and generate optimal forecast as well as various associated statistics. These algorithms are arranged hierarchically to perform the following sequence of operations.

(1) Optional autocorrelation, partial autocorrelation, and cross-correlation functional analysis permit the user to scan for evidence of

nonstationarity in the time series and to identify dynamic relationships between series. These statistics provide reasonable clues to time series nonstationarity, as evidenced by non-declining autocorrelation and partial autocorrelation values (Box and Jenkins, 1970). But only a rough approximation of the underlying dynamic relationships between series can be expected from cross-correlational analysis, as cross correlations are inefficient estimates confounded by autocorrelation effects within series. The Nonstationary time series may be rendered stationary by a difference or a logarithmic transformation. These transformations are effected by calling the data entry module.

(2) Assuming stationarity, the parameters of an autoregressive vector model are first estimated by linear least-squares method.

(3) The autoregressive vector estimates are then transformed into nonlinear autoregressive-moving average vector parameter estimates via an inverse function. The inverse function links the linear autoregressive parameters to linear combinations of nonlinear autoregressive-moving average parameters.

(4) These transformed nonlinear estimates are then entered as initial values into a nonlinear least-squares estimation routine. These values start off the iterations in the nonlinear estimation method which eventually leads to the maximum likelihood estimates.

(5) Next, autocorrelations and partial autocorrelations of the residual are derived for the fitted model, and cross-correlations between pairs of residual series and actual observations are computed. These statistics should reveal any remaining stochastic trends within each time series or any causal relationships left unexplained by the model. In addition, chi-square tests of

significance are performed to differentiate significant trends from random fluctuations. When no significant trends are revealed by the autocorrelations, by the partial autocorrelations, and by the cross-correlations, the fitted model is adequate.

(6) Parameter estimates from the fitted model are transmitted to a main forecasting program which, in turn, determines the form of forecasts to be generated. This main program coordinates several forecasting algorithms.

   (a) A general algorithm which generates multiple series of forecasts on the basis of past autoregressive-moving average dynamics and of the leading indicators. Moreover, the algorithm adjusts for all difference operations performed on the original series by re-incorporating them into the forecasts.

   (b) An algorithm computes the variance of the forecasts and constructs the confidence bands for each forecast.

   (c) An adaptive forecasting algorithm updates existing forecasts by adjusting for the forecast error revealed by each new piece of information entering the system.

   (d) An algorithm that combines forecasts generated by alternative methods to arrive at a single series of forecasts.

   (e) An algorithm computes the standard normal probability associated with each forecast.

4.0 System Testing

Tests of validity and forecast reliability are implemented at each step of CEWMFC system construction. Each subroutine is designed to achieve a

certain performance level and is tested at each step of system development. The test employed is known as "sensitivity analysis" (De Neufville and Stafford, 1971). It gauges how a change in one part of the system effects the whole. By this method either we vary the parameter values or we vary the input series and inspect the corresponding changes in the output series for evidence of possible instability.

Moreover, upon completion of the entire system, a test of forecast reliability is performed with data secured within the system.

4.1 Forecast Reliability Test

The test, known as "retrospective forecasting", gauges the historical accuracy of the forecasts. By this method, a segment of the latest data is compared with forecasts for the same time period. Large deviations of the forecasts from the actual observations would cast doubt on the reliability of the forecast system.

Since random disturbances are always present in the forecasts, discrepancies are inevitable. In order to differentiate systematic from randomly induced deviations of forecasts from actual observations, a significance test is introduced. The test Statistics, devised by G.E.P. Box and G. Tiao (1977) is: $Q = \underset{\sim}{e}'\underset{\sim}{V}^{-1}\underset{\sim}{e}$ which is distributed as chi squares with m degrees of freedom, and where $\underset{\sim}{e}$ is a 1 x m vector of forecast errors defined as the difference between the actual observations and the forecasts, i.e., $\underset{\sim}{e} = \underset{\sim}{Z} - \underset{\sim}{Z}(\ell)$ and $V^{-1} = \underset{\sim}{\Psi}'\underset{\sim}{\Psi}/\sigma^2$, where $\underset{\sim}{\Psi}$ is an m x m matrix of coefficients defining the relationships between time series white noise processes and forecast errors generated from the same process, e.g., $\underset{\sim}{a}_t = \Psi\underset{\sim}{e}_t$. The test addresses the null hypothesis that the observed deviations are results of random disturbances and not indicative of systematic errors.

This test has a major drawback, for it uses the same data that guided the formulation of the model to test forecast accuracy. The result is a tendency to understate the amount of forecast errors and thus bias the outcome in favor of confirming forecast reliability.

4.2 Hypotheses

For testing purpose, indicators are constructed specifically for the CEWMFC system. These indicators include various political crisis and economic/defense policy measures. The selection criteria draw heavily upon current social science theories which may be stated in hypothetical form.

Two hypotheses underlie the selection of indicators. These hypotheses adopt the factional model in elite analysis. By definition elites manipulate power, and power or inter-factional power distribution is the focal point of our model. The model analyzes factional groupings to determine how behavioral characteristics and career mobility patterns of faction members are associated with political crises and policy outcomes. Briefly, these hypotheses may be stated as follows.

H_1: The frequency of purges and demotions increases during periods of political crises and displays patterns of collective experience shared by members of elite factions set apart by competing interests and value orientations.

H_2: Changes in leadership resulting from factional differences lead to significant readjustments in policy priorities.

Underlying these hypotheses are the assumptions that politics in China since 1949 has been largely dominated by factional power struggles and that while policy differences do play a role they serve more often as the pretexts for rivalry than as the causes of conflict. Nevertheless, once a conflict is resolved the dominant faction re-determines the policy priorities. The logic behind these assumptions is developed as follows.

With "politics in command" (Gray, 1974) of all policy decisions regarding production, the distribution of wealth and authoritative allocation of values,

differences invariably will emerge to divide the elites along diverging lines of interests and value orientations. Along these cleavages, elite factions tend to coalesce about natural alliances based on shared background attributes and common career experiences (Chang, 1969). From such collective experiences, there emerge the personal ties essential for the working of factional politics. Nathan (1973) has observed that the factions, principally at the central governmental level, form the bases through which personal ambitions are fostered and political power accrued. As the power balance tips in favor of particular factions, groups of elites share collective experiences of career successes and failures. Thus, a recent Rand study (Sung, 1975) found that during crisis periods, elites with certain shared interests and background attributes are more likely to have similar occupational mobility patterns than those who do not.

Since elite factions are defined along the lines of existing power cleavages, the historical twists and turns of their relative power positions offer illuminating insights into trends of power distribution in Chinese politics. As the power position of each faction depends largely on the extent of control its members can exercise on important party and governmental decision-making organs, the rate of positional loss such as demotion has a profound effect on prevailing power distribution. While a normal rate of personnel loss resulting from natural causes is inevitable, a high turnover rate in an otherwise stable elite group is indicative of changing factional power balance. Accordingly, Oksenberg (1974) has found in his analysis of the Chinese elite "exit pattern" that a high rate of purges and demotions is the usual pattern of personnel attrition in times of crises. As major power redistribution among factions invariably disturbs the leadership it creates stresses in the political system and threatens to alter the existing

political objectives and policy priorities. The resultant uncertainty is likely to cause a cessation of normal activities in the political process. Thus, while the relative personnel loss rates among factions are not direct measures of political crises, they are nonethless good proxy indicators.

Since power redistribution among factions in the leadership often leads to policy changes, it carries profound policy implications for Chinese politics. Most great movements of the recent past, the Great Leap Forward of 1958-1960, the Cultural Revolution of 1966-1969, and the purge of the "radical four" in 1976, have been marked by high levels of personnel losses, followed closely by major policy changes. Tracing the broad configurations of historical evidence, scholars have fashioned various explanations of Chinese politics. While all have observed some oscillations in patterns of policy changes some have attributed those patterns to impacts of crises resulting from "left" versus "right" and "idealogue" versus "pragmatist" power struggles (Nathan, 1976). Parris Chang (1975), for example, notes that policy shifts in China are often associated with changes in the power balance among contending "left-right" factions in the decision-making councils.

The linkage between political crises and policy changes is supported by a rich heritage of historical and empirical evidence (Rosenau, 1969; Liao, 1976; Nathan, 1976). In domestic policies, one may trace a clear chronology of parallel movements in crisis developments and policy shifts. For example, the Great Leap Forward of 1958-1960 initiated an era of mass-mobilization culminating in policies of collectivization and rapid industrialization. The excesses of the Great Leap then led to a period of retrenchment towards decentralization and tolerance of material incentives lasting from the late 1950's to 1965. With the coming of the Cultural Revolution in 1965, all pragmatic policies associated with planning,

managerialism, specialization and technology were cast to the wind, to be superceded by ideological dogmatism. After four tumultuous years, Chinese policies since 1969 have regained some degree of moderation -- in spite of a brief interlude of radicalism in the mid-1970's -- and redound to a search for pragmatic solution. These events, among other historical and empirical evidence, supply substance to a linkage between crises and policy changes in China.

In short, it is suggested by the above discussion that (a) factions embody conflicting interests and value orientations and the relative rates of personnel losses between factions provide good indications of political crises, and (b) political crises induce changes in Chinese military and economic policies.

4.3 Data

Data are collected for various Chinese political crisis and economic/military policy indicators. For the crisis indicator a measure of elite turnover rates is used. The turnover rates are measured in terms of the frequency of purges and demotions.

Although there is no totally satisfactory way of identifying factions, one possibility is to define factions along biographical attributes that imply shared values and interests. For example, the following attributes are selected for this study.

-commander versus commissar career patterns

-military versus civilian career patterns

-administrative "functional" affiliations

-military region and field army affiliations

-generational affiliations

As defined along these biographical attributes the relative turnover rates within the factions reveal the pattern of inter-factional conflict signaling the onset of crisis.

Approximately 568 top level party and governmental personnel are selected for the analysis, including members of the Politburo and of the Central Committee, ministers of the state Council, commanders and first political commissars of the military regions and the service arms, and the governors and first party secretaries in the provinces and on the revolutionary committees. Of these, certain biographical data are available for 400 elites as compiled in a previous Rand elite biographical study covering the period 1956 to 1973 (Sung, 1975).* Our analysis thus extends the data collection effort through 1978.

Since the Central Committee has undergone substantial changes in membership since 1974, 86 new members have been identified. For these elites we have also collected various other biographical data which are not used in testing the present system but which are nonetheless necessary for maintaining some consistency in extending the earlier data set. These attributes obtained for the new members are:

-place and date of birth,

-education, military and civilian,

-combat experience,

-awards,

-date entered party.

Annual data for the policy indicators are collected along different economic and military dimensions from 1956 to 1978. They include:

-gross national product,

-export and import levels,

*Two cases, Yang Ching-jen and Chiang Wei-ching, were each inappropriately coded twice as separate individuals in the Rand study. Correcting for these errors reduces our sample size by two.

-total industrial outputs,

-total agricultural outputs,

-defense expenditures.

It should be noted that the quality of Chinese statistics is limited at best. For a combination of reasons, including a traditional lack of concern for numerical accuracy, the manipulation of data for reasons of political expediency, the infancy of the Chinese State Statistical Bureau, the central data collection agency, and the disruptions of the Great Leap forward and the Cultural Revolution to statistical research efforts in China, statistics for most years are not totally accurate and exist for some years only in the form of rough estimates (Orleans, 1974). Nonetheless, the aforementioned data are adequate for the limited present purpose of testing the forecast reliability of the CEWMFC system.

4.4 Indicator Construction

Two sets of indicators are constructed to drive the CEWMFC system.

4.4.1 Crisis Indicators

Several steps are required to construct the crisis indicator. First, we assign our sample of elites to various factions sharing common biographical attributes. Thus we have identified various factions according to their party affiliations, their governmental functional affiliations, their field army affiliations, and their generational affiliations.

Second, we compile the total annual turnover rates due to purges or demotion for the factions.

Third, in order to capture the dynamics of inter-factional conflict, the turnover rates are further transformed into ratio scores. These scores are obtained by juxtaposing competing factions and dividing the turnover

frequency of one into that of the other. For instance,

$$Y_1 = \frac{\text{number of purged and demoted commanders}}{\text{number of purged and demoted commissars}}$$

$$Y_2 = \frac{\text{number of purged or demoted personnel in field army i}}{\text{number of purged or demoted personnel in field army j}}$$

$$Y_3 = \frac{\text{number of purged or demoted personnel in administrative functional area i}}{\text{number of purged or demoted personnel in administrative functional area j}}$$

$$Y_4 = \frac{\text{number of purged or demoted personnel in generation i}}{\text{number of purged or demoted personnel in generation j}}$$

$$Y_5 = \frac{\text{number of purged and demoted civilians}}{\text{number of purged and demoted military personnel}}$$

Finally, because some personnel loss will always occur in any given faction from natural attrition, it is necessary to differentiate crisis rate from the normal loss rate. An index of abnormality, the Z score, is used for this purpose and it is measured according to

$$Z = \frac{Y_t - \bar{Y}_t}{S_y}$$

where $\bar{Y}_t$ is the average score for past Y_t's and S_y the corresponding standard deviation. As Andriole and Young (1977) noted, this score is akin to the standard normal score and it measures the degree of departure of a current value from the past average. A large Z score suggests abnormal departure from a state of "usualness" as the average is taken to imply.

4.4.2 Policy Indicators

The policy indicators are similarly computed in terms of ratio scores to tap the conflict dimension involving competing issues in the policy-making process. Again, juxtaposing measures of competing policies, we take the ratio of their respective scores to produce the following indicators.

$$Y_1 = \frac{\text{industrial outputs}}{\text{agricultural outputs}}$$

$$Y_2 = \frac{\text{defense expenditures}}{\text{gross national product}}$$

$$Y_3 = \frac{\text{total import}}{\text{gross national product}}$$

$$Y_4 = \frac{\text{total export}}{\text{gross national product}}$$

These ratios are intended to reveal the alternating swings between competing policy positions taken by the Chinese leadership between 1956 and 1977.

4.5 System Testing

The CEWMFC system is tested along three dimensions. Sensitivity analysis assesses the stability and the responsiveness of the system to parameter changes, input variations, and effect of perturbations in subcomponents on the overall system. Forecast reliability test ascertains the accuracy of the forecasts generated by the system. Finally, hypothesis testing tests the ability of the system to discern complex network of interrelationships to generate the corresponding parameter estimates, and to provide multiple forecasts.

4.5.1 Sensitivity Analysis

Sensitivity analyses at every level of the CESMFC system reveal the system on the whole to be stable. With further explication provided by "help" statements, the system prompts user commands with direct requests for specific inputs. The choices of inputs requested are displayed on screen, thus restricting the user to a limited range of operations. All unacceptable commands are screened out by the interface routines, and the correct commands are suggested by help statements and error messages. This has the effect of insulating the system from inappropriate command inputs.

The database manager is, in contrast, totally insulated from external inputs. No direct user access is permitted into the manager, and since the manager also operates independently of data, it is a completely self sufficient unit. As a result, this component of the system is highly stable and functions according to a routine chain of operations. The computational algorithms remain largely stable over a wide range of parameter values. Parameter changes in most subroutines do not adversely affect the performance of the algorithms. This holds true for the linear least-squares routine used in estimating the autoregressive vector model, the inverse function routine, and the various forecasting routines. The exception is the nonlinear least-squares routine for autoregressive-moving average vector model estimation. Unlike other routines, the routine is sensitive to parameter perturbations. Although a superior modification of the gradient search method, the Fletcher-Powell optimization algorithm used in the nonlinear least-squares routine is limited in the number of parameters which it can effectively estimate. While the Fletcher-Powell method is to some extent data dependent, in that model overfitting impairs its estimation efficiency, it has been shown to be capable of accommodating at most five series at a time in lower order vector models. As the order of the model successively increments, the number of iterations required for convergence correspondingly increases. Our experience has shown that in particular for overfitted model with large number of parameters, convergence is difficult to achieve. Many iterations required in computing the optimum step size used in determining the direction vector and in achieving convergence towards the maximum likelihood estimates. Introducing some termination rules, stopping the iterations after a certain number has been reached, has served to hasten convergence. But the effect on the estimation may be negative especially

in situations where the likelihood function is not quadratic and well behaved. Table 1 shows the number of iterations required before convergence convergence is achieved for different orders of models, using one and two series of observations.

Moreover, sensitivity analyses have revealed that due to the limiting numerical capacity (small word size) of the PDP 11/70 minicomputer, the CEWMFC system can not efficiently estimate models with a large number of parameters. Testing with five series at a time often results in floating overflows at several points of the system. The determinants of matrices, such as the residual sum-of-squares-and-cross-product matrices, for higher order ARMAV(m,n) models often assume sizes beyond the limits of the PDP memory word capacity (upper limit of 1.7×10^{38}) causing the system to abort estimation. Writing the program in double precision form has ameliorated the problem but has also doubled the space requirement of the system in relation to single precision format. Furthermore, setting upper bounds to limit determinant size has also been attempted. Such bounds prevent the system from aborting in extreme cases of large determinant values but they also impair the system's ability to generate maximum likelihood estimates. Nonetheless, since large determinants of residual matrices often are indicative of ill fitted models, which should be eliminated from consideration by the model search routine, generating inefficient estimates for those models should not affect the selection and estimation of the final optimal model. The optimal model will in all likelihood produce small residual values.

Moreover, use of difference operation to remove nonstationarity in time series is highly recommended. Estimation with nonstationarity series seldom leads to rapid convergence and less frequently to efficient estimates. When the nonstationary parameters have values close to 1 differencing has the added advantage of reducing the number of parameters that have to be estimated.

Table 1

NUMBER OF ITERATIONS FOR CONVERGENCE
OF DIFFERENT ORDERS OF ARMAV(m,n) MODELS

Series	Orders (1,0)	(2,0)	(1,1)	(2,1)	(3,2)	(4,3)
Chinese commander/ commissar						
No. of iterations	1			27	50	73
F Test significance level	.05			.05	.05	.05
German Naval Expenditure						
No. of iterations	1	22	51	35		
F Test significance level	.05	--	.05	--		
Iterative Anglo-German Naval Expenditures						
No. of iterations	41	50	51	82		
F Test significance level	.05	--	--	--		

This would hasten convergence and lessen the likelihood of overflowing system capacity. Testing using different data sets, including the Chinese crisis and policy indicator series, monthly Wisconsin employment series, annual Anglo-German naval expenditure series (1870-1914) have revealed the system to be particularly sensitive to changes in nonstationary series. It is thus recommended that all series entered into the system be stationary or be rendered stationary by means of the difference or the logarithmic transformation in the system.

Similarly, the system may be sensitive to changes in time series when the number of observations is small. Since in short series, each observation can be expected to contribute more to determine the nature of the data distribution than would an observation in a lengthier series, minor variations in a few observations can be expected to affect and alter the forecast outcomes. Increasing the number of observations in time series should help to lessen system sensitivity to minor variations in time series.

Finally, the system appears to operate best with lower order models in which the number of parameters is limited. Tests of Chinese elite data reveal that the system functions most efficiently with less than five series. Convergence towards maximum likelihood values is rapid and forecasts most precise with single series of observations. The system operates with relative efficiency for two and three series, but encounters more difficulty as it advances towards estimating five or more series. Because of the large number of parameters involved in multivariate time series models each successively higher order increases the number of parameters substantially, (e.g., for models with five series increment of each order adds 25 parameters to be estimated: ARV(1) contains 25 parameters, ARMAV(1,1) contains 50 parameters). Consequently, as the number of time series increases the order of models which the CEWMFC

system can effectively estimate becomes progressively smaller. Tables 2a to 2d show the inverse relationship between the orders of models and the number of series.

Tables 2a through 2d also illustrate the automated sequence of operations performed by the CEWMFC system as it attempts model fitting. Table 2b, for example, shows that the system first examines and compares progressively higher orders of mainline models from which it identifies ARMAV(2,1) to give the optimal fit. Then, comparing this model to alternative intermediary models, it found only ARMAV(3,1) model to yield significant improvement in relation to ARMAV(2,1). Since, there are several statistically significant models, the system attempts a further forecast mean-square-error test to single out the best forecast model. The results favor retention of ARMAV(3,1) as the final forecast model.

4.5.2 Forecast Reliability Test

The forecast accuracy of the CEWMFC system is assessed by means of a retrospective forecasting method. The method, as described in section 4.1, evaluates the performance of the system by the amount of forecast errors incurred. The errors are obtained by comparing retrospective forecasts for past periods with corresponding known observations. The forecast errors are then examined for systematic, nonrandom discrepancies. Large discrepancies in forecast errors are indicative of a lack of reliability in the forecast results.

Results of retrospective forecasting and the attendant statistical significance tests for two separate time series are displayed in Tables 3a and 3b. The columns of the tables display the actual observations, the ℓ-step-ahead forecasts from a fixed time origin, the associated forecast

errors, the one-step-ahead forecasts generated from successive time origins, the noise a_t's which are equivalent to the one-step-ahead forecast errors, and the squared a_t's. The displays also include the variances of a_t and the chi-square Q statistics used to determine the statistical significance of the forecast errors.

The test results indicate that the forecasts are generally in agreement with the actual observations in terms of magnitude and of direction of movement over time. Greater accuracy seems to characterize the forecasts from the employment series than from the Chinese industry/agriculture indicator series. The former generates smaller forecast errors and its movement more closely monitors and reflects the movement of the actual observations, whereas the latter follows less closely the movement of the actual observations. A comparison of the forecast errors reveals that while no distinguishable pattern of errors is discernible in the employment forecasts, forecast errors for the Chinese series do get larger over time. These divergent results arise in part from the fact than the employment series has five times as many observations as the Chinese indicator series, which would have contributed to more efficient modeling and estimation and thus to more accurate forecasts in the former.

Table 2a

Automated Model Fitting Output
for One Series

Y_t = industry/agriculture outputs

Model	F Test	location
ARMA(1,0)	significant	mainline
ARMA(2,1)	not significant	mainline
ARMA(1,1)	significant	intermediary

Forecast mean-square error test reveals best model to be ARMA(1,1)

Final model = ARMA(1,1)

AR(1) = .62679
MA(1) =-.71071

Table 2b

Automated Model Fitting Outputs
for Two Series

Y_{1t} = import/GNP

Y_{2t} = generation 1/generation 2

Model	F Test	location
ARMAV(1,0		mainline
ARMAV(2,1)	significant	mainline
ARMAV(3,2)	not significant if	mainline
ARMAV(2,2)	not significant if	intermediary
ARMAV(3,1)	significant	intermediary
ARMAV(2,0)	not significant if	intermediary
ARMAV(1,1)	not significant if	intermediary

mean-square-error reveal best model is ARMAV(3,1)

ARMAV(3,0)	not significant if	test for zero moving average

Final model = ARMAV(3,1)

$$\begin{bmatrix} 2.0232 & -.29534 \\ .51865 & .031225 \end{bmatrix} \text{ARV(1)} \qquad \begin{bmatrix} 1.033 & -1.1852 \\ 1.2205 & .59642 \end{bmatrix} \text{MA(1)}$$

$$\begin{bmatrix} -1.8559 & -.078638 \\ -.61496 & .45622 \end{bmatrix} \text{ARV(2)}$$

$$\begin{bmatrix} .71867 & -.94981 \\ .52372 & .30357 \end{bmatrix} \text{ARV(3)}$$

Table 2c

Automated Model Fitting Outputs for Three Series

Y_{1t} = industrial/agricultural output

Y_{2t} = export/GNP

Y_{3t} = import/GNP

Models	F Tests	location
ARMAV(1,0)	significant	mainline
ARMAV(2,1)	significant	mainline
ARMAV(3,2)	not significant	mainline
ARMAV(2,2)	not significant	intermediary
ARMAV(3,1)	not significant	intermediary
ARMAV(1,1)	not significant	intermediary
ARMAV(2,0)	not significant	intermediary and lower order moving average

Comparison of forecast mean-square-errors of significant models reveal ARMAV(2,1) to yield best forecast.

Final model = ARMAV(2,1)

(The CEWMFC system then employs the final model for forecasting purpose).

$$\begin{array}{c|ccc} & Y_{1t-1} & Y_{2t-1} & Y_{3t-1} \\ \hline Y_{1t} & .65716 & -.36788 & .21350 \\ Y_{2t} & -.66100 & -.10157 & 3.7925 \\ Y_{3t} & -1.2828 & .00938 & 2.6144 \end{array}$$

$$\begin{array}{c|ccc} & Y_{1t-2} & Y_{2t-2} & Y_{3t-2} \\ \hline Y_{1t} & .58059 & -.35158 & .40317 \\ Y_{2t} & -.26823 & .11792 & -2.3294 \\ Y_{3t} & -.63255 & .14310 & .80377 \end{array}$$

$$\begin{array}{c|ccc} & a_{1t-1} & a_{25-1} & a_{3t-1} \\ \hline Y_{1t} & .68273 & -.69998 & .74970 \\ Y_{2t} & .85698 & .59884 & .20334 \\ Y_{3t} & 2.0397 & .80272 & -.40041 \end{array}$$

Table 2d

Automated Model Fitting Output

for Four Series

Y_{1t} = industry/agriculture

Y_{2t} = import/GNP

Y_{3t} = export/GNP

Y_{4t} = civilian/military

Model	F Test	location
ARMAV(1,0)	significant	mainline
ARMAV(2,1)	not significant	mainline
ARMAV(1,1)	not significant	intermediary
ARMAV(2,0)	significant	

Forecast mean'square test reveals best model to be ARMAV(1,0)

Final Model = ARMAV(1,0)

$$\begin{array}{c|cccc} & Y_{1t-1} & Y_{2t-1} & Y_{3t-1} & Y_{4t-1} \\ \hline Y_{1t} & -.90788 & .26891 & -.03007 & .048038 \\ Y_{2t} & 2.4836 & .27589 & .51484 & .45307 \\ Y_{3t} & 1.4570 & .57049 & .76769 & .32054 \\ Y_{4t} & -1.7077 & -.0377 & .19815 & .13675 \end{array}$$

Table 3a

Forecast Reliability Test
Wisconsin Employment Series
n=104

steps ahead	observations	ℓ-step-ahead forecasts	forecast errors	one-step-ahead forecasts	a^*_t	a_t^2
Wisconsin series n=104						
1	550.0	548.35	1.7	549.15	.849	.721
2	544.9	538.15	6.8	539.87	5.026	25.262
3	542.4	533.45	9.0	541.22	1.175	1.380
4	539.0	533.88	5.1	541.73	-2.735	7.485
5	532.5	528.46	4.1	529.78	2.512	6.314
6	532.5	528.46	4.8	533.52	-.323	.104
7	533.5	531.37	2.2	536.07	-2.578	6.648
Sum					3.926	47.914

$$S^2 = [\Sigma at^2/n] - [(\Sigma a_t)^2/n^2] = 6.53$$

$$Q = 1/S^2 \; \Sigma a_t = 1/6.53 \;\; 47.91 = 7.9$$

Since chi-square critical level is $X^2_{.05,7} = 14.07$, test suggests no systematic errors

* the a_t's are the one-step ahead forecast errors computed from

$$a_t = Y_{t+1} - Y_t(1)$$

where $Y_t(1)$ is the one-step-ahead forecast from time origin t corresponding to the actual observation Y_{t+1}.

Table 3b

Forecast Reliability Test
Chinese Industry/Agriculture series
n=13

step ahead	observations	ℓ-step-ahead forecasts	forecast errors	one-step-ahead forecasts	a^*_t	a_t^2
1	3.3899	3.1965	.1934	3.196	.193	.0372
2	3.9199	3.0269	.8830	2.931	.988	.9761
3	4.2799	3.0953	1.1846	2.973	1.306	1.7051
Sum					2.467	2.7184

$$S^2 = [\Sigma a_t{}^2/n] - [(\Sigma a_t)^2/n^2] = .2299$$

$$Q = 1/S^2 \ \Sigma \ a_t{}^2 = 1/.2299 \times 2.7184 = 11.787$$

Since chi-square critical level is $X^2_{.05,3} = 7.81$, test suggest presence of systematic error.

*the a_t's are the one-step ahead forecast errors computed from

$$a_t = Y_{t+1} - Y_t(1)$$

where $Y_t(1)$ is the one-step-ahead forecast from time origin t corresponding to the actual observation Y_{t+1}.

Finally, tests of significance with Q statistics reveal no systematic errors in the employment forecasts, suggesting that the observed errors might have been induced by random disturbances. In contrast, test results do indicate that some systematic errors do exist in the Chinese indicator forecasts. The associated Q statistic rejects the null hypothesis that the observed deviations between the forecasts and the actual observations are randomly induced.

Based on the above results, we must conclude that as of the date of the writing of this report there is still an absence of consistency in the forecast performance of the CEWMFC system. Further efforts are recommended and underway to improve the forecast accuracy of the system.

4.5.3 Data Analysis

Analysis of data with the CEWMFC system has produced some evidence in support of the hypotheses that

(1) The frequency of demotion increases during periods of political crises and displays patterns of collective experience shared by members of competing factions.

(2) Changes in leadership resulting from factional differences lead to adjustments in policy priorities.

Continuing the trends observed in the previous Rand study (Sung, 1975), the post-1974 data reveal high demotion frequencies coinciding with known crisis periods. In particular, personnel demotion rate is highest in 1977 coinciding with the crisis attendant to the political demise of the "Gang of Four".

As tables 4a to 4e show, the total number of demotions in any one of the factional categories for that year is at least twive as large as that for any other year.

Moreover, significant differences also appear in the relative demotion frequencies between factions. High demotion frequencies appear to concentrate in particular factions. Among factions, defined according to functional affiliations, the highest demotion frequencies occur among military service corps (e.g., armor, artillery, infantry, public security, etc.) as compared to much lower frequencies among personnel in civil administrative functional areas. This reflects the continued effort to lessen military presence in the governmental and party hierarchy in the post-Lin Piao era. Among the field army factions, high purge frequencies are observed in the first, third, fourth, and fifth field armies. It is significant to note that the frequency of personnel loss is high in the 4th army with which Lin Piao was closely identified. In contrast, the 2nd field army faction continues to enjoy its post-Lin Piao era dominance observed in the earlier Rand study. It has the lowest demotion frequency of all the field armies. Likewise, a pattern of concentration also appears among the generational factions. Higher demotion rate seems to plague the older generations, such as generations one, two, and three, as contrasted to the relatively low frequencies in generations four and five. Thus, beyond personnel loss resulting from shifting power distribution, the foregoing results also show the slow but steady generational replacement effect at work. Moreover, civilian and military leaders have also fallen collectively as groups since 1974. With the exception of 1976, the power position of the military in relation to the civilian elites has undergone serious erosion since 1974. The pattern displayed is in line with the trend towards waning military influence in party and government in the post Lin Piao era.

And consistent with earlier findings (Sung, 1975) no distinguishing patterns of behavior separate the relative demotion frequencies between commanders and commissars.

Tests of hypothesis H_2 employing the models outlined in Appendix C have yielded some evidence of relationship between crisis occurrence and policy changes. A sample of the structural estimates for various models is displayed in Tables 5a to 5l. The stimates of crisis impacts on policies are located in the verticle rectangular enclosures in the five series models and on the upper right-hand corner of the parameter matrices for the two series models, whereas estimates of the reverse effects are located in the horizontal rectangular enclosures in the five series models and on the lower-left hand corner of the parameter matrices of the two series models. Most of these estimates are significantly different from zero suggesting the existence of causal relationships between the crisis and the policy indicators. For example, the relative swings of policy preference between industrial and agricultural development seem to reflect the relative power positions of generation one and two and of civilian-military relations. Likewise, the patterns of external trade relations, measured in terms of the proportion of import and export to total GNP, also seem to respond to the sequence of crisis events. The crisis indicators, measured in terms of demotion frequencies by functions, by generation, by field army, by civil-military and by commander-commissar distinctions, all appear to exert strong impacts on China's level of foreign trade. Confirming some earlier findings (Liau, 1976) these results give further credence to the suspected influence of domestic politics on China's relations with the external world. In short, our data reveal that political crises do have significant policy consequences in China.

Furthermore, there is also some limited evidence of a reciprocal effect of policy differences on inter-factional disputes. For instance, estimates in Tables 5a to 51 suggest that the ratios of industrial to agricultural outputs, of import and export to GNP are causally antecedent to relative demotion frequencies in factions defined according to generational differences, functional differences, civilian-military distinction, and field army affiliations. These results suggest the possibility of policy differences fueling existing inter-factional disputes. According to the power model this reciprocal effect would occur as factions highlight their policy differences to camouflage more basic differences over the distribution of power and political resources. Nevertheless, differences over policies, issues and ideology do serve to heighten the fundamental disputes over power distrubtion. Yet, without more solid evidence, we can only tentatively entertain this power interpretation that political crises, which result in changes in leadership, carry significant policy impacts and often contribute to policy changes, even though policy differences might have initally exacerbated the inter-factional disputes that led to the outbreak of crisis. While a more precise interpretation of our data must await further analysis, our findings have not been contradicted by the recent developments in China.

Table 4a

DISTRIBUTION OF DEMOTIONS BY FUNCTION

	Function[a]													
Year	01	02	03	04	05	06	07	08	09	10	11	12	UNK	TOT
*1974	0	0	0	1	0	0	0	0	0	0	1	0	0	2
1975	2	12	1	1	0	0	0	0	2	0	0	0	7	25
1976	1	5	0	1	1	0	2	5	3	0	1	0	2	22
1977	15	17	0	5	1	1	0	2	2	0	0	0	38	81
1978	5	3	1	2	0	0	0	0	2	1	0	0	15	29

Table 4b

DISTRIBUTION OF DEMOTIONS BY FIELD ARMY

Year	FA1	FA2	FA3	FA4	FA5	FA6	UNK	TOT
*1974	0	0	0	1	1	0	0	2
1975	1	1	6	5	7	4	1	25
1976	12	1	8	10	7	2	2	41
1977	16	9	13	13	15	12	3	81
1978	4	6	6	5	3	2	3	29

*The 1974 data are underestimates of the true demotion frequencies. A large number of demotion frequencies for 1974 were not considered in the present sample. As the previous Rand study contained data up to September, 1973 and since our present analysis began in January 1974, all cases in between were left out. Hence, until the missing data are accounted for our 1974 estimates must be treated with extreme caution.

Table 4c

DISTRIBUTION OF DEMOTIONS BY GENERATION

	Actual						
Year	G1	G2	G3	G4	G5+	UNK	TOT
*1974	1	0	1	0	0	0	2
1975	6	3	8	1	1	6	25
1976	3	4	7	1	2	24	41
1977	11	14	15	1	3	39	81
1978	3	4	6	1	0	15	29

Table 4d

DISTRIBUTION OF DEMOTIONS BY COMMISSARS VS. COMMANDERS

	Actual				
Year	CSR	CDR	BOTH	UNK	TOT
*1974	0	1	0	1	2
1975	4	11	0	16	25
1976	8	6	0	27	41
1977	18	18	0	45	81
1978	7	8	0	16	29

Table 4e

DISTRIBUTION OF DEMOTIONS BY CIVILIAN VS. MILITARY

	Actual			
Year	CIV	MIL	UNK	TOT
*1974	1	1	0	2
1975	4	21	0	25
1976	23	15	3	41
1977	33	43	5	81
1978	12	17	0	29

Table 5a

Structural Model of Crisis and Policy Behavior

Series:	$\bar{Y}_i$	σ_i^2
Y_1 = industrial outputs/agricultural outputs	2.28696	.92357
Y_2 = military expenditures/GNP	.05087	.00364
Y_3 = imports/GNP	13.39696	25.2821
Y_4 = exports/GNP	14.20087	21.046
Y_5 = function 10/function 2	.14130	.05824

Final model = ARMAV(1,0)

Parameters: ARV(1)

$$
\begin{array}{c|ccccc}
\text{dependent} \backslash \text{independent} & Y_1 & Y_2 & Y_3 & Y_4 & Y_5 \\
\hline
Y_1 & .61038 & .26485 & -.85671 & .94770 & -.043456 \\
Y_2 & .22138 & .84007 & 1.0581 & -1.0528 & -.088777 \\
Y_3 & .35984 & -.029768 & .20999 & .59037 & -.17143 \\
Y_4 & .25930 & -.04606 & .097177 & .84520 & -.19545 \\
Y_5 & .40321 & -.13835 & 1.9833 & -1.8214 & -.50717
\end{array}
$$

Table 5b
Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_1$	σ_1^2
Y_{1t} = industrial/agricultural	2.2869	.92357
Y_{2t} = military/GNP	.05087	.00364
Y_{3t} = imports/GNP	13.396	25.046
Y_{4t} = exports/GNP	14.2000	21.046
Y_{5t} = commander/commissar	.57348	.34040

Final model = ARMAV(1,0)

Parameters: ARV(1)

dependent	Y_{1t-1}	Y_{2t-1}	Y_{3t-1}	Y_{4t-1}	Y_{5t-1}
			independent		
Y_{1t}	-.88452	.07566	.16749	5.7859	.78736
Y_{2t}	-.16591	.67830	.10168	.92992	-.35042
Y_{3t}	1.3248	.24062	4.0957	-2.3769	-11.367
Y_{4t}	.63098	.89301	.93337	7.9989	.71657
Y_{5t}	.75224	.00521	2.2212	1.2577	7.2970

Table 5c

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_1$	σ_1^2
Y_{1t} = industrial/agricultural	2.2896	.92337
Y_{2t} = military/GNP	.05087	.00364
Y_{3t} = imports/GNP	13.39696	25.28210
Y_{4t} = exports/GNP	14.2008	21.046
Y_{5t} = generation 1/generation 2	2.44739	5.22174

Final model = ARMAV(1,0)

Parameters: ARV(1)

dependent	Y_{1t-1}	Y_{2t-1}	Y_{3t-1}	Y_{4t-1}	Y_{5t-1}
			independent		
Y_{1t}	-.02286	.06271	.55912	.14576	6.6459
Y_{2t}	.01711	.00288	.07275	.03432	.91595
Y_{3t}	.61437	-.14851	2.6791	15.848	1.0845
Y_{4t}	6.3405	.05100	.85618	-12.203	4.7928
Y_{5t}	111.95	1.2116	18.623	237.26	88.451

Table 5d

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = industrial/agricultural	2.28696	.92357
Y_{2t} = military/GNP	0.05087	.00364
Y_{3t} = imports/GNP	13.39696	25.8210
Y_{4t} = exports/GNP	14.20087	21.046
Y_{5t} = army 3/army 4	1.13609	1.47215

Final model = ARMAV(1,0)

Parameters: ARV(1)

	independent				
dependent	Y_{1t-1}	Y_{2t-1}	Y_{3t-1}	Y_{4t-1}	Y_{5t-1}
Y_1	.69385	.37355	-.95428	1.0762	-.032068
Y_2	.30945	.47646	.99309	-1.2580	.14412
Y_3	.57728	-.06476	.28414	.25812	.077751
Y_4	.38408	.35024	-.14481	.88775	.026938
Y_5	-.91837	.34083	.37253	.10855	-.16016

Table 5e
Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_1$	σ_1^2
Y_{1t} = industrial/agricultural	2.2869	.92357
Y_{2t} = military/GNP	.05087	.00364
Y_{3t} = imports/GNP	13.396	25.2821
Y_{4t} = exports/GNP	14.2008	21.046
Y_{5t} = army 5/army 3		.48729

Final model = ARMAV(1,0)

Parameters: ARV(1)

$$
\begin{array}{c c}
 & \text{independent} \\
\text{dependent} & \begin{array}{ccccc} Y_{1t-1} & Y_{2t-1} & Y_{3t-1} & Y_{4t-1} & Y_{5t-1} \end{array} \\
\begin{array}{c} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{array} &
\begin{bmatrix}
-.00536 & .05234 & .35064 & 5.6371 & .08555 \\
-.04116 & .01768 & .08462 & .87402 & -.55823 \\
.57089 & -.51382 & 2.3937 & 11.387 & -21.727 \\
2.3304 & -.35605 & 1.0416 & -.03542 & -2.9381 \\
.50778 & 1.0819 & 2.5977 & 22.564 & 44.494
\end{bmatrix}
\end{array}
$$

Table 5f
Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = industrial/agricultural	2.2869	.92357
Y_{2t} = military/GNP	.05087	.00364
Y_{3t} = imports/GNP	13.396	25.2821
Y_{4t} = exports/GNP	14.2008	21.046
Y_{5t} = civilian/military	1.55652	4.83198

Final model = ARMAV(1,0)

Parameters: ARV(1)

dependent	independent Y_{1t-1}	Y_{2t-1}	Y_{3t-1}	Y_{4t-1}	Y_{5t-1}
Y_{1t}	.02019	.04374	.58995	-.42051	5.6929
Y_{2t}	-.11889	-.12550	.08889	.26181	.84583
Y_{3t}	.46054	.41365	2.8545	-7.7956	5.4345
Y_{4t}	4.9622	.11104	.51701	-10.952	.74756
Y_{5t}	3.9102	-.49414	.93132	31.092	67.421

Table 5g

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = industrial/agriculture	2.28696	.92357
Y_{2t} = army 3/army 4	1.13609	1.47215

Final Model = ARMAV(2,1)

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .10862 & -.43470 \\ .54098 & -.15756 \end{bmatrix}} \end{array}$$

$$\begin{array}{ccc} & Y_{1t-2} & Y_{2t-2} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .18493 & -.14847 \\ -.73590 & -.11882 \end{bmatrix}} \end{array}$$

$$\begin{array}{ccc} & a_{1t-1} & a_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -.87145 & -.53762 \\ 2.836 & -.58577 \end{bmatrix}} \end{array}$$

Table 5h

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = industry/agriculture outputs	2.28696	.92357
Y_{2t} = generation 1/generation 2	2.44739	5.22174

Final Model = ARMAV(2,1)

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .77864 & .15278 \\ -2.3840 & -.81341 \end{bmatrix}} \end{array}$$

$$\begin{array}{ccc} & Y_{1t-2} & Y_{25-2} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .24029 & .05737 \\ -1.3712 & -.19182 \end{bmatrix}} \end{array}$$

$$\begin{array}{ccc} & a_{1t-1} & a_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -1.3924 & .007282 \\ -.03714 & -.33059 \end{bmatrix}} \end{array}$$

Table 5i

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ^2_i
Y_{1t} = industry/agriculture	2.28696	.92357
Y_{2t} = civilian/military	1.55652	4.83198

Final Model = ARMAV(2,0)

$$\begin{array}{c} \\ Y_{1t} \\ Y_{2t} \end{array} \begin{array}{c} \begin{array}{cc} Y_{1t-1} & Y_{2t-1} \end{array} \\ \begin{bmatrix} .84853 & .014139 \\ .73531 & .66843 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ Y_{1t} \\ Y_{2t} \end{array} \begin{array}{c} \begin{array}{cc} Y_{1t-1} & Y_{2t-1} \end{array} \\ \begin{bmatrix} .065049 & .08041 \\ -1.069 & -.46098 \end{bmatrix} \end{array}$$

Table 5j

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = imports/GNP	13.396	25.2821
Y_{2t} = army 3/army 4	1.13609	1.47215

Final Model = ARMAV(2,1)

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-2} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} 1.0275 & .51491 \\ -.20356 & -.13655 \end{bmatrix}} \end{array} \quad \text{ARV(1)}$$

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -.36262 & .49502 \\ .18451 & -.19940 \end{bmatrix}} \end{array} \quad \text{ARV(2)}$$

$$\begin{array}{ccc} & a_{1t-1} & a_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -.75691 & -.072273 \\ -.63184 & -1.2647 \end{bmatrix}} \end{array} \quad \text{MAV(1)}$$

Table 5k

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = import/GNP	13.39896	25.8210
Y_{2t} = civilian/military	1.55652	4.83198

Final model = ARMAV(2,1)

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .56450 & 1.7151 \\ -.53944 & 1.0247 \end{bmatrix}} \end{array} \quad \text{ARV(1)}$$

$$\begin{array}{ccc} & Y_{1t-2} & Y_{2t-2} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} .52554 & .17885 \\ .65635 & .08087 \end{bmatrix}} \end{array} \quad \text{ARV(2)}$$

$$\begin{array}{ccc} & a_{1t-1} & a_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -1.111 & 3.1178 \\ -.69913 & .07573 \end{bmatrix}} \end{array} \quad \text{MAV(1)}$$

Table 51

Structural Model of Crisis and Policy Behavior

Series	$\bar{Y}_i$	σ_i^2
Y_{1t} = export	14.2008	21.046
Y_{2t} = army 3/army 4	1.13609	1.47215

Final model = ARMAV(2,1)

$$\begin{array}{ccc} & Y_{1t-1} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} 1.4945 & .14815 \\ -.54953 & -.08204 \end{bmatrix}} \end{array} \quad \text{ARV(1)}$$

$$\begin{array}{ccc} & Y_{1t-2} & Y_{2t-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -.72985 & -.29755 \\ .46952 & -.45482 \end{bmatrix}} \end{array} \quad \text{ARV(2)}$$

$$\begin{array}{ccc} & a_{2t-1} & a_{25-1} \\ \begin{array}{c} Y_{1t} \\ Y_{2t} \end{array} & \multicolumn{2}{c}{\begin{bmatrix} -.09623 & 1.8497 \\ -1.1739 & .7785 \end{bmatrix}} \end{array} \quad \text{MAV(1)}$$

APPENDIX A

Technical Descriptions of the CEWMFC System

A.1.0 Autoregressive-Moving Average Vector Time Series Model

The "autoregressive-moving average vector" (ARMAV) model in this software describes a system of feedback relations within the framework of an autoregressive-moving average process. All variables are assumed to be interdependent over time, so that the system encompasses a range of structural relationships. Nonetheless, a parsimonious description of the system is possible through vector representation.

Consider a system in which the state variables at time t are dependent on variables at preceding times, t-m through t-1, and on their corresponding noise series from times t-n to t-1. Then a representative autoregressive-moving average model would be an ARMAV (m,n) process:

$$\underset{\sim}{Y}_t = \underset{\sim}{\phi}_1 \underset{\sim}{B}^d \underset{\sim}{Y}_{t-1} + \ldots + \underset{\sim}{\phi} \underset{\sim}{B}^d \underset{\sim}{Y}_{t-m} + \underset{\sim}{a}_t - \underset{\sim}{\theta}_1 \underset{\sim}{B}^d \underset{\sim}{a}_{t-1} - \ldots - \underset{\sim}{\theta}_n \underset{\sim}{B}^d \underset{\sim}{a}_{t-n} \qquad \text{(A.1-1)}$$

where,

$$\underset{\sim}{\phi}_1 = \begin{bmatrix} \phi_{111} & \phi_{121} & \ldots & \phi_{1p1} \\ \phi_{211} & \phi_{221} & \ldots & \phi_{2p1} \\ \vdots & \vdots & & \vdots \\ \phi_{p11} & \phi_{p21} & & \phi_{pp1} \end{bmatrix} \quad \ldots \quad \underset{\sim}{\phi}_m = \begin{bmatrix} \phi_{11m} & \phi_{12m} & \ldots & \phi_{1pm} \\ \phi_{21m} & \phi_{22m} & \ldots & \phi_{2pm} \\ \vdots & \vdots & & \vdots \\ \phi_{p1m} & \phi_{p2m} & & \phi_{ppm} \end{bmatrix}$$

$$\underset{\sim}{\theta}_1 = \begin{bmatrix} \theta_{111} & \theta_{121} & \ldots & \theta_{1p1} \\ \theta_{211} & \theta_{221} & \ldots & \theta_{2p1} \\ \vdots & \vdots & & \vdots \\ \theta_{p11} & \theta_{p21} & \ldots & \theta_{pp1} \end{bmatrix} \quad \ldots \quad \underset{\sim}{\theta}_n = \begin{bmatrix} \theta_{11n} & \theta_{12n} & \ldots & \theta_{1pn} \\ \theta_{21n} & \theta_{22n} & \ldots & \theta_{2pn} \\ \vdots & & & \\ \theta_{p1n} & \theta_{p2n} & \ldots & \theta_{ppn} \end{bmatrix}$$

$$\underset{\sim}{Y}_t = \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ \vdots \\ Y_{pt} \end{bmatrix} \quad \ldots \quad \underset{\sim}{Y}_{t-m} = \begin{bmatrix} Y_{1t-m} \\ Y_{2t-m} \\ \vdots \\ Y_{pt-m} \end{bmatrix} \qquad \underset{\sim}{a}_t = \begin{bmatrix} a_{1t} \\ a_{2t} \\ \vdots \\ a_{pt} \end{bmatrix} \quad \ldots \quad \underset{\sim}{a}_{t-n} = \begin{bmatrix} a_{1t-n} \\ a_{2t-n} \\ \vdots \\ a_{pt-n} \end{bmatrix}$$

$$
\underset{\sim}{B}^{d} = \begin{bmatrix}
(1-B^{s_1})^{d_{11}}(1-B)^{d_{12}} & 0 & \cdots & 0 \\
0 & (1-B^{s_2})^{d_{21}}(1-B)^{d_{22}} & \cdots & 0 \\
0 & 0 \quad . & \cdots & 0 \\
\vdots & \vdots & . & \vdots \\
0 & 0 \quad \cdots & . & (1-B^{s_p})^{d_{p1}}(1-B)^{d_{p2}}
\end{bmatrix}
$$

s_i = seasonal operator of indicator i

d_{i1} = number of seasonal differences for indicator i

d_{i2} = number of regular differences for indicator i

The subscripts in the above equations denote the row and column position of elements in the system of equations and the order of the autoregressive or moving average process respectively. In each of the above matrices, $\underset{\sim}{\phi}$ or $\underset{\sim}{\theta}$, the diagonal elements, $\underset{\sim}{\phi}_{iij}$ or $\underset{\sim}{\theta}_{iik}$ (for $i=1,\ldots,p$, $j=1,\ldots,m$ and $k=1,\ldots n$) contain the autoregressive and moving average parameters for each series, i.e., ϕ_{111} defines the dependence of Y_{1t} on Y_{1t-1}, ϕ_{11m} of Y_t on Y_{1t-m}, θ_{11n} of Y_t on a_{1t-1}, and θ_{11n} of Y_t on a_{1t-n}. The off-diagonal elements, in contrast, determine the inter-relationships between variables. For instance ϕ_{121} defines the dependence of Y_{1t} on Y_{2t-1}, ϕ_{12m} the dependence of Y_{1t} on Y_{2t-m}, ϕ_{211} the dependence of Y_{2t} on Y_{1t-1}, ϕ_{21m} of Y_{2t} on Y_{1t-m}. A similar interpretation applies to the off-diagonal elements in the $\underset{\sim}{\theta}$ matrices. Elements in the $(I - B)^d$ matrices are difference operators (i.e. $(1-B)Y_t = Y_t - Y_{t-1}$). Use of these operators in time series models is one way of eliminating nonstationary in time series. Systems so differenced will have characteristic roots that lie within the unit circle. The condition for stationarity in the vector model will be that the eigenvalues of the $\underset{\sim}{\phi}$ and $\underset{\sim}{\theta}$ matrices lie within the unit circle. This condition will be periodically checked in our modeling procedure.

Each element in $\underset{\sim}{a}_t$ is assumed to be normally distributed with mean zero and variance σ_a^2. Moreover, $\underset{\sim}{a}_t$ also satisfies the conditions that

(1) $\underset{\sim}{a}_t$ is independent of $\underset{\sim}{a}_{t-1}, \ldots, \underset{\sim}{a}_{t-n}$

(A.1-2)

(2) $\underset{\sim}{a}_t$ is independent of $\underset{\sim}{Y}_{t-m-1}, \ldots, \underset{\sim}{Y}_{t-m-k}$

In other words, no reverse dependence of $\underset{\sim}{a}_t$ on past $\underset{\sim}{a}_t$'s or on past $\underset{\sim}{Y}_t$'s beyond the autoregressive order is permitted. Furthermore, since $\underset{\sim}{Y}_t$'s coefficient matrix is the identity, no contempomneous relationships are assumed to exist among the elements of $\underset{\sim}{Y}_t$ (e.g., $\underset{\sim}{Y}_{1t}, \ldots, \underset{\sim}{Y}_{pt}$). Finally, no restriction is imposed on the dependence of Y_{it} on the past a_{jt}'s, for $i \neq j$ (For instance, the dependence of Y_{1t} on a_{2t-1}).

Moreover, by imposing certain restrictions on elements of the vector one might also entertain various alternative special cases. For example, the existing system is reducible to a single equation autoregressive-moving average process:

$$Y_t = \phi_1 Y_{t-1} + \phi_m Y_{t-m} + a_t - \theta_1 a_{t-1} - \ldots - \theta_n a_{t-n} \qquad \text{(A.1-3)}$$

Although other models are reducible from (3.1-3), further analysis of their likelihood function is necessary before we may implement them in our forecasting software.

A.2.0 Modeling Strategy: Fitting Autoregressive-Moving Average Vector Model

Our modeling strategy encompasses a three-step procedure. The steps are identification, estimation, and diagnostic checking. In identification, we select that model which best fits the observed time series. In estimation, we derive estimates of the model parameters. In diagnostic checking, we check the adequacy of the estimated model. Inadequate models are re-identified and re-estimated until adequacy is assured.

Moreover, identification, estimation, and diagnostic checking are performed within a hypothesis testing framework. By setting up pairs of successive orders of models under null and alternative hypotheses, we can test alternative choices of parameter restrictions. Through successive comparison of higher order models we will be able to incrementally approximate the observed time series and determine the best combination of autoregressive-moving average forms.

A.2.1 Model Identification

Employing a hypothesis testing approach, vector model identification calls for the fitting and comparison of pairs of successively higher orders of models until all of the systematic variations in the time series are accounted for. Accordingly, successive orders of ARMAV (m,n) models are fitted until a point is reached beyond which no statistically significant improvement in model approximation can be expected. The last model to register a significant improvement is retained as the optimal or best fitting model. This procedure is outlined in the following steps.

(1) The first question to be posed in model fitting is: what is the optimal order of the ARMAV (m,n) model? In other words, what are the values of m and n? This question is best answered by examining successive values of m and n until a model is reached that no longer displays significant improvement in goodness of fit (this criterion will be defined later). This approach presupposes a systematic and natural progression in the sizes of m and n examined. Indeed,

it has been shown that any stationary process can be adequately approximated by increasing orders of an ARMAV (m,m-1) model for large enough m (Pandit, 1973). Accordingly we introduce a modeling strategy based on the following sequence of steps:

a. Beginning with ARMAV(1,0) we increment our model by (m,m-1) order, where m=1,2,.... Along this progression, we would successively examine higher order models, such as ARMAV(1,0), ARMAV(2,1), ARMAV(3,2), ARMAV (4,3),..., ARMAV(m,m-1). The increments are by order of 1 for both the autoregressive and the moving average components. We terminate this progression when no further improvement in goodness of fit can be expected from proceeding to a higher order model.

b. Since the models are incremented by orders of (m,m-1), there are alternative intermediary models not accounted for by the above procedure. These are models resulting from incrementing the m or n value separately by order of 1. These alternative possibilities to each ARMAV (m,m-1) model are shown in the boxes enclosed in dotted lines as shown in Figure 3. Within each box we compare the alternative intermediary models to the mainline ARMAV (m,m-1) models to determine the optimal model according to some statistical criteria of goodness of fit. However, we do not examine all the boxes, but only the box corresponding to the best ARMAV (m,m-1) model. Thus we begin by successively incrementing the autoregressive and the moving average components in ARMAV (m,m-1) model by orders of 1 until we arrive at a mainline model beyond which no further improvement can be expected. We then test the alternative intermediary models in the corresponding box against the final ARMAV (m,m-1) model, and determine the optimal choice. For instance, we might first proceed along the

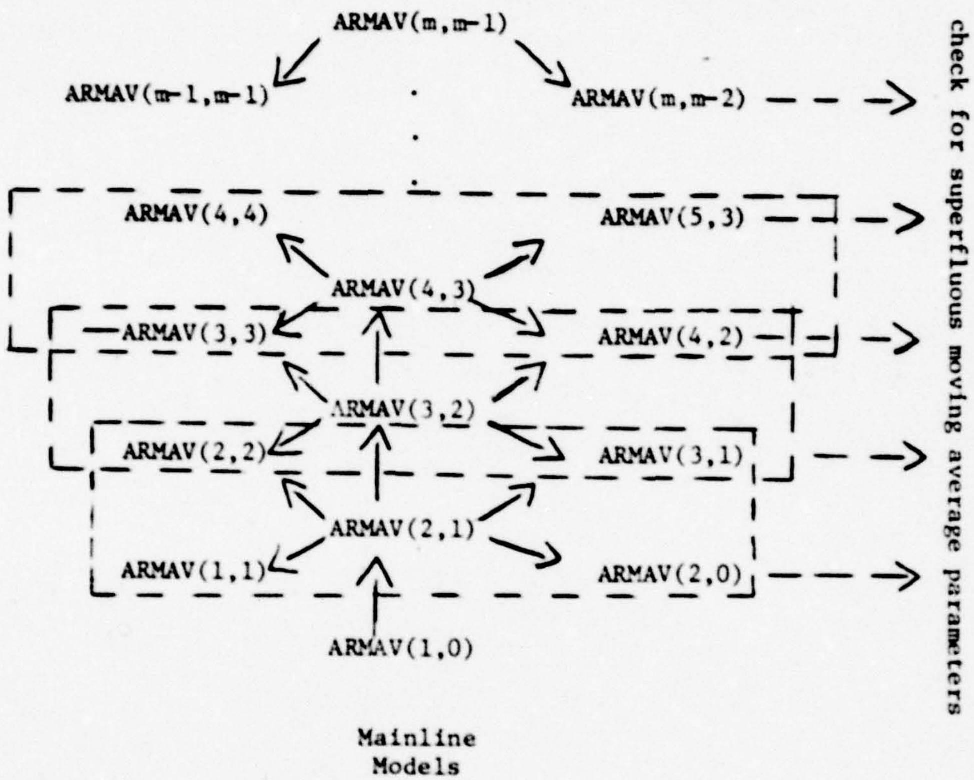
mainline models (e.g., ARMAV(1,0), ARMAV(2,1), ARMAV(3,2), ARMAV(4,3)) and find that the no significant improvement in goodness of fit results from going beyond ARMAV(3,2) to ARMAV(4,3). We would thus drop back to ARMAV(3,2) and test it against its corresponding intermediary models enclosed in the same box. By focusing on the intermediary models in the same box, namely, ARMAV(3,3), ARMAV(4,2), ARMAV(2,2), and ARMAV(3,1), we have successfully bypassed suboptimal lower order intermediary models. The saving in terms of computational effort is obvious.

c. As a final test of model adequacy, we check for superfluous moving average parameters in the final model selected in the preceeding step. In this test, we successively decrease the order of the moving average parameters until no superfluous parameters remain, as a substantial change in model goodness of fit, due to the dropping of parameters, would indicate. Accordingly, the final check involves testing in declining order a sequence of models: ARMAV(m,n-2), ARMAV(m,n-3),..., ARMAV(m,0). (Note that this final step is not necessary if the optimal model selected at step b is ARMAV(m,m-1), since its corresponding lower order moving average models also constitute the intermediary cases already examined).

Methodologically, this approach adopts a regression perspective. It is predicated on the assumption that the dependency in the time series can principally be modeled by autoregressive processes. The moving average processes merely account for the remaining stochastic dynamics in the series. Thus, by this reasoning it is entirely possible to couple a lower order vector moving average with an ARV(m) component.

Figure 3

ARMAV(m,n) Modeling Strategy



(2) Selection of model is based on a goodness of fit criterion (smallest sums-of-squares of the residual) by which we judge the adequacy of a model in capturing the systematic trends in the time series. The goodness of fit criterion is applied through a hypothesis testing approach. Accordingly, we examine a pair of models at a time in increasing order to compare the observed difference in their goodness of fit and to determine whether the differences are results of random errors or of some systematic effects. The higher order model is known as the full model and the lower order model as the restricted model. Parameters which appear in the full model but not in the restricted model are assumed to have been restricted. Applying the hypothesis testing approach, we would set up a null hypothesis to correspond to the restricted model and an alternative hypothesis to correspond to the full model. The null hypothesis defines the restricted parameters as zeros and the alternative hypothesis sets them to some non-zero values. Thus in comparing a restricted ARMAV(2,1) model,

$$\underline{Y}_t = \underline{\phi}_1 \underline{Y}_{t-1} + \underline{\phi}_2 \underline{Y}_{t-2} + a_t - \underline{\phi}_1 \underline{a}_{t-1} \qquad \text{(A.2-1)}$$

to a full ARMAV(3,2) model,

$$\underline{Y}_t = \underline{\phi}_1 \underline{Y}_{t-1} + \underline{\phi}_2 \underline{Y}_{t-2} + \underline{\phi}_3 \underline{Y}_{t-3} + \underline{a}_t - \underline{\phi}_1 \underline{a}_{t-1} - \underline{\phi}_2 \underline{a}_{t-2} \qquad \text{(A.2-2)}$$

our null and alternative hypothesis would be respectively,

$$\begin{aligned} H_N&: \ \underline{\Theta} = \underline{\phi}_2 = 0\underline{I} \\ H_{A}&: \ \underline{\phi}_3 \neq 0I_{\underline{2}} \neq 0\underline{I} \end{aligned} \qquad \text{(A.2-3)}$$

(3) A statistical test of significance serves to distinguish between the full and the restricted model. The choice of model is based on two alternative tests. They are the F test and a test involving comparison of forecast mean-square-errors. We describe these tests briefly below.

(a) The F test determines whether the observed differences in the models' residual sum-of-squares, which measures the models' goodness of fit, indicate true improvement in model adeouacv or are attributable to random effects. The F statistic used is (Rao, 1973, pp. 555)

F Tests for Alternative p and s

Values of p,s	F ratio test	degrees of freedom
s=1 for any p	$\dfrac{(1-\Lambda)/p}{\Lambda/(N-r+s-p)}$	p and N-r+s-p
s=2 for any p	$\dfrac{(1-\sqrt{\Lambda})/p}{\sqrt{\Lambda}/(N-r+s-p-1)}$	2p and 2(N-r+s-p-1)

(A.2-4)

where

r = number of parameters in the full ARMAV(m,n) model = $(m + n)p^2$

s = number of parameter matrices restricted to zero in the restricted model

p = number of variables or series in the vector model

N = number of observations in each time series.

$$\Lambda = \frac{|A_A|}{|A_N|} = \frac{|a_A' a_A|}{|a_N' a_N|}$$

The $|A_N|$ and $|A_A|$ are determinants of the quadratic residual sum-of-squares-and-product matrices (i.e., $A_N = a_N' a_N$ where a is an nx1 vector of noise series containing elements $a_{1t}, \ldots, a_{nt}$) for the restricted model (under the null hypothesis) and the full model (under the alternative hypothesis) respectively:

b. Forecast mean-square-error comparison permits us to determine the relative forecast performance among any number of models. According to this test, the model with the least forecast mean-square-error has the best forecast performance. We use for this test a statistic known as the "Thiel coefficient" (Thiel, 1965):

$$MSE = \bar{e}^2 + S_Y^2 + S_y^2 - 2\, S_Y S_y\, r(Y_t, y_t) \qquad \text{(A.2-5)}$$

where Y_t is the time series observations and $y_t = Y_t\,(\ell)$ their corresponding forecasts,

$e_t = Y_t - y_t$, and

$$\bar{e}^2 = \left(\frac{\sum_{t=1}^{N} e_t}{N}\right)^2$$

$$S_Y = \sqrt{\frac{\sum_{t=1}^{N} Y_t^2 - \left(\sum_{t=1}^{N} Y_t\right)^2 / N}{N}}$$

$$S_y = \sqrt{\frac{\sum_{t=1}^{N} y_t^2 - \left(\sum_{t=1}^{N} Y_t\right)^2 / N}{N}}$$

$$r_{Y,y} = \sqrt{\frac{\sum_{t=1}^{N} Y_t y_t - \left(\sum_{t=1}^{N} Y_t\right)\left(\sum_{t=1}^{N} y_t\right)}{\left[N \sum_{t=1}^{N} Y_t^2 - \left(\sum_{t=1}^{N} Y_t\right)^2\right]^{\frac{1}{2}} \left[N \sum_{t=1}^{N} y_t^2 - \left(\sum_{t=1}^{N} y_t\right)^2\right]^{\frac{1}{2}}}}$$

Note that N is the number of observations in each series; $\bar{e}^2$, S_Y, S_y and $r_{Y,y}$ are the square of the averaged forecast errors, the standard deviation of Y_t, the standard deviation of y_t, and the correlation coefficient between Y_t and y_t respectively.

By this test criterion, we would select as the optimal model the one giving the least forecast mean-square-error measured in terms of the Thiel coefficient.

(4) The foregoing tests are used alternately in different phases of our modeling strategy. Within the hypothesis testing framework the F statistic is used initially in selecting the best mainline ARMAV(m,m-1) models and then is used in identifying the "significant" intermediary models localized about the best ARMAV(m,m-1) model within a given box in Figure 3. Having identified the significant models we next compare the estimated Thiel coefficients for each model to determine the model which gives the best forecast results. Finally, we test for superfluous moving average parameters in the optimal model by comparing its forecast mean-square-error with those of models having successively lower order moving average components. This procedure continues until no superfluous parameters are left. The model that we ultimately retain is therefore both significant and has the least amount of forecast errors.

A.2.2 Model Estimation

Model estimation entails an automated three-step procedure. In step one, model fitting is accomplished by linear least-squares. The CEWMFC system automatically fits data to successively higher orders of autoregressive models. In step two, a transformation, through an "inverse function", of the autoregressive parameters generates initial estimates of nonlinear autoregressive-moving average vector model parameter. These estimates are needed to start off the iterations in nonlinear least-squares estimation. In step three, final parameter estimates are derived for the autoregressive-moving average vector model. Using the estimates from the inverse function as initial values, a nonlinear least-squares routine, based on a Fletcher-Power minimization algorithm, fine tunes the initial values until they are near maximum likelihood.

A.2.2.1 Estimating ARV(m) Linear System (ARVEST)

Estimation of a pure autoregressive vector model may be accomplished by a linear least-squares method. Consider, for instance, the model

$$\underset{\sim}{Y}_t = \underset{\sim}{\phi}\underset{\sim}{X}_t + \underset{\sim}{a}_t \tag{A.2-6}$$

where Y_t, X_t, and a_t are 1xp vectors; and $\underset{\sim}{\phi}$ is a pxp matrix of coefficients. To estimate the $\underset{\sim}{\phi}$ matrix we minimize the sum of squares and cross-product matrix for $\underset{\sim}{a}_t$:

$$\begin{aligned} \underset{\sim}{A} = \underset{\sim}{a}_t'\underset{\sim}{a}_t = E\ (\underset{\sim}{Y}_t - \underset{\sim}{\phi}\underset{\sim}{X}_t)'\ (\underset{\sim}{Y}_t - \underset{\sim}{\phi}\underset{\sim}{X}_t) \\ = \underset{\sim}{Y}_t'\underset{\sim}{Y}_t - \underset{\sim}{Y}_t'\underset{\sim}{X}_t - \phi'\underset{\sim}{X}_t'\underset{\sim}{Y}_t + \phi'\underset{\sim}{X}_t'\underset{\sim}{X}_t\ \phi \\ = \underset{\sim}{Y}_t'\underset{\sim}{Y}_t - 2\phi'\underset{\sim}{X}_t'\underset{\sim}{Y}_t + \underset{\sim}{\phi}'\underset{\sim}{X}_t'\underset{\sim}{X}_t\underset{\sim}{\phi} \end{aligned} \tag{A.2-7}$$

Differentiating $\underset{\sim}{A}$ with respect to ϕ yields.

$$\frac{\partial A}{\partial \phi} = -2\underset{\sim}{X}_t'\underset{\sim}{Y}_t + 2\underset{\sim}{X}_t'\underset{\sim}{X}_t \tag{A.2-8}$$

Setting the derivative equal to zero and substituting $\hat{\phi}$ for ϕ, we get the normal equation

$$\underset{\sim}{X}_t'\underset{\sim}{X}_t\ \hat{\underset{\sim}{\phi}} = \underset{\sim}{X}_t'\underset{\sim}{Y}_t \tag{A.2-9}$$

from which the least-squares estimate obtained is

$$\hat{\underset{\sim}{\phi}} = (\underset{\sim}{X}_t'\underset{\sim}{X}_t)^{-1}\underset{\sim}{X}_t'\underset{\sim}{Y}_t \tag{A.2-10}$$

In estimating the vector model

$$\underset{\sim}{\phi}_0\ \underset{\sim}{Y}_t = \underset{\sim}{\phi}_1\underset{\sim}{Y}_{t-1} + \dots + \underset{\sim}{\phi}_m\underset{\sim}{Y}_{t-m} \tag{A.2-11}$$

we set X_t as a partitioned matrix:

$$\underset{\sim}{X}_t = [\underset{\sim}{Y}_t \vdots \underset{\sim}{Y}_{t-1} \vdots \cdots \vdots \underset{\sim}{Y}_{t-m}]$$

and regress $\underset{\sim}{Y}_t$ on $\underset{\sim}{X}_t$ as shown above. Setting $\underset{\sim}{\phi}_1^* = \underset{\sim}{\phi}_1\underset{\sim}{\phi}_0^{-1}, \dots, \underset{\sim}{\phi}_m^* = \underset{\sim}{\phi}_m\underset{\sim}{\phi}_0^{-1}$, we get the linear ARV(m) estimates.

A.2.2.2 Estimating Initial Values for ARMAV(M,n) Model (INVEST)

The initial values for the ARMAV(m,n) model may be derived by making use of a simple relationship between a mixed autoregressive-moving average process and a pure autoregressive process. It is known (Box and Jenkins, 1970) that a mixed ARMAV(m,n) process, expressed in terms of backshift operators, $\underset{\sim}{B} = (B_1,\ldots,B_p)$,

$$(\underset{\sim}{I} - \underset{\sim}{\phi}_1\underset{\sim}{B} - \ldots - \underset{\sim}{\phi}_m\underset{\sim}{B}^m)\underset{\sim}{Y}_t = (\underset{\sim}{I} - \underset{\sim}{\theta}_1 B - \ldots - \underset{\sim}{\theta}_n\underset{\sim}{B}^n)\underset{\sim}{a}_t \qquad \text{(A.2.12)}$$

has an equivalent "inverse" function,

$$\underset{\sim}{a}_t = (I - \underset{\sim}{\pi}_1\underset{\sim}{B} - \underset{\sim}{\pi}_2\underset{\sim}{B}^2 - \ldots)\underset{\sim}{Y}_t \qquad \text{(A.2.13)}$$

where the $\underset{\sim}{\pi}$'s where the linear combinations of the $\underset{\sim}{\phi}$'s and $\underset{\sim}{\theta}$'s.

The above relation is established by substituting the pure model for $\underset{\sim}{a}_t$ in the mixed model, which gives us an operator identity (after cancelling out $\underset{\sim}{Y}_t$ from both sides of the equation):

$$(\underset{\sim}{I} - \underset{\sim}{\phi}_1\underset{\sim}{B} - \ldots - \underset{\sim}{\phi}_m\underset{\sim}{B}^m) = (\underset{\sim}{I} - \underset{\sim}{\theta}_1\underset{\sim}{B} - \ldots - \underset{\sim}{\theta}_n\underset{\sim}{B}^n)\ (\underset{\sim}{I} - \underset{\sim}{\pi}_1\underset{\sim}{B} - \underset{\sim}{\pi}_2\underset{\sim}{B}^2 - \ldots)$$

Equating coefficients of equal powers of $\underset{\sim}{B}$ (see Li, 1979; Box and Jenkins, 1970) we get the following system of equalities

$$\begin{aligned}
\underset{\sim}{\phi}_1 &= \underset{\sim}{\theta}_1 + \underset{\sim}{\pi}_1 \\
\underset{\sim}{\phi}_2 &= \underset{\sim}{\theta}_2 - \underset{\sim}{\theta}_1\underset{\sim}{\pi}_1 + \underset{\sim}{\pi}_2 \\
\underset{\sim}{\phi}_3 &= \underset{\sim}{\theta}_3 - \underset{\sim}{\theta}_2\underset{\sim}{\pi}_1 - \underset{\sim}{\theta}_1\underset{\sim}{\pi}_2 + \underset{\sim}{\pi}_3 \\
&\vdots \\
\underset{\sim}{\phi}_j &= \underset{\sim}{\theta}_j - \underset{\sim}{\theta}_1\underset{\sim}{\pi}_{j-1} - \underset{\sim}{\theta}_2\underset{\sim}{\pi}_{j-2} - \cdots - \underset{\sim}{\theta}_{j-1}\underset{\sim}{\pi}_1 + \underset{\sim}{\pi}_j
\end{aligned} \qquad \text{(A.2.14)}$$

In particular, for j greater than either of the larger orders of m and n, we have

$$(\underset{\sim}{I} - \underset{\sim}{\theta}_1\underset{\sim}{B} - \underset{\sim}{\theta}_2\underset{\sim}{B}^2 - \ldots - \underset{\sim}{\theta}_n\underset{\sim}{B}^n)\ \underset{\sim}{\pi}_j = 0 \qquad \text{(A.2-15)}$$

It should be apparent from the above expressions that the initial guess values for the $\underset{\sim}{\phi}$'s and the $\underset{\sim}{\theta}$'s can be derived once the $\underset{\sim}{\pi}$'s are known. Since the inverse

function is an infinite order autoregressive process, it can be approximated by a vector autoregressive process:

$$a_t = (\underset{\sim}{I} - \underset{\sim}{\beta}_1 B - \dots - \underset{\sim}{\beta}_k B^k) Y_t \tag{A.2-16}$$

setting

$$\underset{\sim}{\beta}_j = \underset{\sim}{\pi}_j \qquad , j = 1, \dots, k$$

and defining k = m+n, we obtain our estimates for the inverse function. This equality holds so long as the moving average parameters are invertible, that is, if the moving average component converges. This requires that the eigenvalues ε_i of the $\underset{\sim}{\theta}$ matrices lie between -1 and 1. When this condition is not satisfied by the data at hand we may impose convergence on the moving average vector component by inverting the eigenvalues that are larger than 1, i.e. $1/\varepsilon_i$, to assure compliance with the invertibility condition. Since the above vector model is linear we may estimate the $\underset{\sim}{\beta}$'s by a linear least squares method

$$\underset{\sim}{\beta} = (\underset{\sim}{X}_t' \underset{\sim}{X}_t)^{-1} \underset{\sim}{X}_t' \underset{\sim}{Y}_t \tag{A.2.17}$$

where $\underset{\sim}{X}_t = [\underset{\sim}{Y}_t \mid \underset{\sim}{Y}_{t-1} \mid \dots \mid \underset{\sim}{Y}_{t-k}]$.

Having estimated $\underset{\sim}{\beta}_j$ we next equate it to $\underset{\sim}{\pi}$. Substituting the $\underset{\sim}{\pi}$'s into (A.2-15) we derive the $\underset{\sim}{\theta}_j$'s for $j > m$ (m being the order of the autoregressive vector). Substituting the estimated $\underset{\sim}{\pi}_j$ and $\underset{\sim}{\theta}_j$ into (A.2.14) we can directly derived our initial estimates for the $\underset{\sim}{\phi}$'s.

Consider, for example, an ARMAV(2,1) model:

$$\underset{\sim}{Y}_t = \underset{\sim}{\phi}_1 \underset{\sim}{Y}_{t-1} + \underset{\sim}{\phi}_2 \underset{\sim}{Y}_{t-2} + \underset{\sim}{a}_t - \underset{\sim}{\theta}_1 \underset{\sim}{a}_{t-1} \tag{A.2.18}$$

In order to derive the initial estimates for $\underset{\sim}{\phi}_1$, $\underset{\sim}{\phi}_2$, and $\underset{\sim}{\theta}_1$, we must first approximate the corresponding inverse function. Since there are three matrices of parameters in the above equation, the inverse function would be

$$\underset{\sim}{Y}_t = \underset{\sim}{\pi}_1 \underset{\sim}{Y}_{t-1} + \underset{\sim}{\pi}_2 \underset{\sim}{Y}_{t-2} + \underset{\sim}{\pi}_3 \underset{\sim}{Y}_{t-3}$$

for $k = 2+1 = 3$. Using linear least squares method we derive the linear approximation for $\underline{B}_1$, $\underline{B}_2$, and $\underline{B}_3$. Then, equating $\underline{B}_j = \underline{\pi}_j$ and using (A.2-15) for $j > 2$, we derive our initial estimate of $\underline{\theta}_1$ as follows,

$$(\underline{I} - \underline{\theta}_1 \underline{B})\, \underline{\pi}_j = 0 \qquad , \text{ for } j > 2$$

which is

$$(\underline{I} - \underline{\theta}_1 \underline{B})\, \underline{\pi}_3 = 0$$

or

$$\underline{\pi}_3 - \underline{\theta}_1 \underline{\pi}_2 = 0$$

Solving the above equation for $\underline{\theta}_1$ we get

$$\underline{\theta}_1 = \underline{\pi}_3 \underline{\pi}_2^{-1} \tag{A.2-19}$$

Next, we check for invertibility. Invertibility requires that the eigenvalues of $\underline{\theta}_1$ lie within the unit circle, -1 and 1. If any eigenvalue ϵ_i, exceeds the upper and lower limits we would invert it. (e.g., $\epsilon_i^* = 1/\epsilon_i$). We compute the eigenvalues using the RTFIND routine. We substitute the ϵ_i^* for the invertible eigenvalues and reintroduce the results into the following expression

$$\underline{\theta}_1^* \underline{T} = \underline{\epsilon}^* \underline{T} \tag{A.2-20}$$

where $\underline{T}$ is the pxp matrix of eigenvectors corresponding to the original untransformed eigenvalues, and $\underline{\epsilon}^*$ is a pxp diagonal matrix containing inverted eigenvalues. $\underline{\theta}_1^*$ is the new coefficient matrix to be estimated from $\underline{T}$ and $\underline{\epsilon}^*$. Transposing matrices in the above expression we may obtain the new matrix:

$$\underline{\theta}_1^* = \underline{T}\underline{\epsilon}^* \underline{T}^{-1}$$

Since $\underline{\epsilon}^*$ contains only invertible elements $\underline{\theta}_1^*$ would satisfy the invertibility condition. This step will be applied to the $\underline{\theta}$ matrix even when it contains no invertible eigenvalues.

Knowing $\underline{\theta}_1$, $\underline{\pi}_1$, $\underline{\pi}_2$, and $\underline{\pi}_3$, we can then calculate $\underline{\phi}_1$ and $\underline{\phi}_2$.

Using (3.2-12) we get

$$\underline{\phi}_1 = \underline{\theta}_1 + \underline{\pi}_2$$

$$\underline{\phi}_2 = \underline{\theta}_2 - \underline{\theta}_1\underline{\pi}_1 + \underline{\pi}_2 \qquad \text{(A.2-21)}$$

Accordingly, for an ARMAV(2,1) model containing two equations, let the least squares estimates from an ARV(3) model be

$$\underline{\beta}_1 = \underline{\pi}_1 = \begin{bmatrix} .200 & .700 \\ -.100 & -.800 \end{bmatrix}$$

$$\underline{\beta}_2 = \underline{\pi}_2 = \begin{bmatrix} .720 & 1.210 \\ .520 & -.010 \end{bmatrix}$$

$$\underline{\beta}_3 = \underline{\pi}_3 = \begin{bmatrix} .256 & .607 \\ .424 & .359 \end{bmatrix}$$

Noting that $\underline{\pi}_2^{-1} = \begin{bmatrix} .015 & 1.902 \\ .817 & -1.131 \end{bmatrix}$

$$\underline{\theta}_1 = \underline{\pi}_3\,\underline{\pi}_2^{-1} = \begin{bmatrix} .256 & .607 \\ .424 & .359 \end{bmatrix} \begin{bmatrix} .015 & 1.902 \\ .817 & -1.131 \end{bmatrix} = \begin{bmatrix} .498 & -.200 \\ .299 & .400 \end{bmatrix}$$

Finding the eigenvalues of $\underline{\theta}_1$

$$\lambda^2 - .898\lambda + .258 = 0$$

$$\lambda_{1,2} = \frac{.898 \pm \sqrt{.806 - 1.032}}{2} = .449 \pm .183i$$

$$|\lambda_{1,2}| = \sqrt{(.449)^2 + (.183)^2} = \sqrt{.234} = .483 < 1$$

We do not need to re-evaluate $\underline{\theta}_1$, since all eigenvalues have magnitude less than 1, and $\underline{\theta}_1^* = \underline{\theta}_1$.

$$\underline{\phi}_1 = \underline{\pi}_1 + \underline{\theta}_1 = \begin{bmatrix} .200 & .700 \\ -.100 & -.800 \end{bmatrix} + \begin{bmatrix} .498 & -.200 \\ .299 & .400 \end{bmatrix} = \begin{bmatrix} .698 & .500 \\ .199 & -.400 \end{bmatrix}$$

$$\underline{\phi}_2 = \underline{\theta}_2 - \underline{\theta}_1\underline{\pi}_1 + \underline{\pi}_2 = \begin{bmatrix} -.498 & .200 \\ -.299 & -.400 \end{bmatrix} \begin{bmatrix} .200 & .700 \\ -.100 & -.800 \end{bmatrix} + \begin{bmatrix} .720 & 1.210 \\ .520 & -.010 \end{bmatrix} = \begin{bmatrix} .601 & .702 \\ .501 & .101 \end{bmatrix}$$

Note that $\underline{\theta}_2$ is assumed to be equal to zero in the last equation.

As another example, consider the following non-stationary case. Given the following output from the ARV routine

$$\Pi_1 = \begin{bmatrix} -.900 & .100 \\ .800 & 1.900 \end{bmatrix} \qquad \Pi_2 = \begin{bmatrix} -.310 & .850 \\ 2.230 & 1.700 \end{bmatrix}$$

$$\Pi_3 = \begin{bmatrix} .687 & 2.805 \\ 3.884 & 2.635 \end{bmatrix}, \text{ then}$$

$$\Pi_2^{-1} = -\frac{1}{2.422} \begin{bmatrix} 1.700 & -.850 \\ -2.230 & -.310 \end{bmatrix} = \begin{bmatrix} -.701 & .350 \\ .920 & -.127 \end{bmatrix}$$

$$\theta_1 = \Pi_3 \Pi_2^{-1} = \begin{bmatrix} .687 & 2.805 \\ 3.884 & 2.635 \end{bmatrix} \begin{bmatrix} -.701 & .350 \\ .920 & .127 \end{bmatrix} = \begin{bmatrix} 2.099 & .596 \\ -.298 & 1.693 \end{bmatrix}$$

The eigenvalues of θ_1 are the solutions to

$$(2.099-\lambda)\ (1.693-\lambda) + (.298)\ (.596) = 0$$

$$\lambda^2 - 3.792\ \lambda + 3.730 = 0$$

$$\lambda_{1,2} = \frac{3.792 \pm \sqrt{14.379 - 14.920}}{2} = 1.896 \pm .367i$$

$$|\lambda_{1,2}| = \sqrt{(1.896)^2 + (.367)^2} = \sqrt{3.728} = 1.930 > 1.0$$

$$\lambda_1 = 1.896 + .367i \qquad 1/\lambda_1 = \varepsilon_1^* = .508 - .098i$$

$$\lambda_2 = 1.896 - .367i \qquad 1/\lambda_2 = \varepsilon_2^* = .508 + .098i$$

The associated eigenvectors for the original eigenvalues are:

$$\lambda_1 \Rightarrow \begin{bmatrix} 1 \\ -.340 + .615i \end{bmatrix} \qquad \lambda_2 \Rightarrow \begin{bmatrix} 1 \\ -.340 - .615i \end{bmatrix}$$

Thus $\underset{\sim}{T} = \begin{bmatrix} 1 & 1 \\ -.340 + .615i & -.340 - .615i \end{bmatrix}$

$$T^{-1} = \frac{1}{.230} \begin{bmatrix} -.340 - .615i & -1 \\ .340 - .615i & 1 \end{bmatrix} = \begin{bmatrix} .500 - .276i & -.813i \\ .500 + .276i & .813i \end{bmatrix}$$

$$\varepsilon^* \quad \begin{bmatrix} .508 - .098i & 0 \\ 0 & .508 + .098i \end{bmatrix}$$

$$T\varepsilon^* = \begin{bmatrix} 1 & 1 \\ -.340 + 615i & -.340 - .615i \end{bmatrix} \begin{bmatrix} .508 - .098i & 0 \\ 0 & .508 + .098i \end{bmatrix}$$

$$= \begin{bmatrix} .508 - .098i & .508 + .098i \\ -.112 + .345i & -.112 - .345i \end{bmatrix}$$

$$\underset{\sim}{\Theta}_1 = \underset{\sim}{T}\,\varepsilon * \underset{\sim}{T}^{-1} = \begin{bmatrix} .508 - .098i & .508 + .098i \\ -.112 + .345i & -.112 - .345i \end{bmatrix} \begin{bmatrix} .500 - .276i & - .813i \\ .500 + .276i & .813i \end{bmatrix}$$

$$\underset{\sim}{\Theta}_1 = \begin{bmatrix} .454 & .158 \\ .078 & -.560 \end{bmatrix}$$

Notice that the eigenvalues of Θ_1 are $\{-.572, .466\}$, with magnitudes less

$$\underset{\sim}{\phi}_1 = \underset{\sim}{\pi}_1 + \underset{\sim}{\Theta}_1 = \begin{bmatrix} -.900 & .100 \\ .800 & 1.900 \end{bmatrix} + \begin{bmatrix} .454 & .158 \\ .078 & .560 \end{bmatrix} = \begin{bmatrix} -.446 & .258 \\ .878 & 1.340 \end{bmatrix}$$

$$\underset{\sim}{\phi}_2 = -\underset{\sim}{\Theta}_1\underset{\sim}{\pi}_1 + \pi_2 = \begin{bmatrix} .454 & .158 \\ .078 & -.560 \end{bmatrix} \begin{bmatrix} -.900 & .100 \\ .800 & 1.900 \end{bmatrix} + \begin{bmatrix} -.310 & .850 \\ 2.230 & 1.700 \end{bmatrix}$$

$$\underset{\sim}{\phi}_2 = \begin{bmatrix} -.282 & .345 \\ -.518 & -1.057 \end{bmatrix}$$

A.2.2.3 Estimating ARMAV(m,n) Nonlinear System (NMLEST)

Estimation of a nonlinear ARMAV(m,n) model is achieved by minimizing the determinant of the sum-of-squares-and-product matrix of $\underset{\sim}{a}_t$ where

$$\underset{\sim}{a}_t = \underset{\sim}{Y}_t - \underset{\sim}{\phi}_1\underset{\sim}{Y}_{t-1} - \cdot\cdot\cdot - \underset{\sim}{\phi}_m\underset{\sim}{Y}_{t-m} + \underset{\sim}{\Theta}_1\underset{\sim}{a}_{t-1} + \cdot\cdot\cdot + \underset{\sim}{\Theta}_n\underset{\sim}{a}_{t-n} \qquad \text{(A.2-22)}$$

in which $\underset{\sim}{Y}_t$'s and $\underset{\sim}{a}_t$'s are px1 vectors; the $\underset{\sim}{\phi}$'s and the $\underset{\sim}{\Theta}$'s are pxp matrices of autoregressive and moving average parameters. The matrix whose determinant is to be minimized is

$$\underset{\sim}{A} = \underset{\sim}{a}_t'\underset{\sim}{a}_t = \begin{bmatrix} \Sigma a_{1t}^2 & \Sigma a_{1t}a_{2t} & \cdots & \Sigma a_{1t}a_{pt} \\ \Sigma a_{2t}a_{1t} & \Sigma a_{2t}^2 & \cdots & \Sigma a_{2t}a_{pt} \\ \vdots & \vdots & \ddots & \\ \Sigma a_{pt}a_{1t} & \Sigma a_{pt}a_{2t} & \cdots & \Sigma a_{pt}^2 \end{bmatrix} \qquad \text{(A.2-23)}$$

with variance-covariance matrix

$$\underset{\sim}{S}_a^2 = 1/n\ \underset{\sim}{A} \qquad \text{(A.2-24)}$$

The method for minimizing $\underset{\sim}{A}$ with respect to the $\underset{\sim}{\phi}$ and $\underset{\sim}{\theta}$ parameters used in this software system is known as the "Fletcher-Powell" method. It is a modified gradient search routine, which, in contrast to the popular steepest descent method that makes use of first derivative information, utilizes second partial derivatives to determine the most direct path by which to move some initial estimates (from the INVEST routine described in the preceding section) towards their maximum likelihood values.

Consider the quadratic function

$$f(\underset{\sim}{Y}) = \underset{\sim}{C}\underset{\sim}{Y} + \underset{\sim}{Y}'\underset{\sim}{D}\underset{\sim}{Y} \qquad \text{(A.2-25)}$$

where C and Y are px1 vectors and D is a pxp matrix and f is a unimodal function. The first derivative at the maxima is therefore

$$f(\underset{\sim}{Y}) = \underset{\sim}{C} + 2\underset{\sim}{D}^{-1}\underset{\sim}{C} \qquad \text{(A.2-26)}$$

which, when set to zero and solved for the optimal point Y*, yields

$$\underset{\sim}{Y}^* = -\tfrac{1}{2}\underset{\sim}{D}^{-1}\underset{\sim}{C} \qquad \text{(A.2-27)}$$

In many practical situations; f(Y) may not be exactly quadratic to provide enough information about C and D to ensure maximum utility of the above technique. But if several points of f are known one might arrive at a useful approximation of (A.2-19) The method thus generates a Hessian matrix of second derivatives for exactly quadratic and unimodal functions but it approximates the Hessian matrix for more complex quadratic functions.

The basic procedure involves the following steps:

(1) Define a positive definite matrix $\underset{\sim}{H}_0$ and some initial values $\underset{\sim}{Y}_0$. For convenience $\underset{\sim}{H}_0$ may be set initially as an identity matrix I.

(2) To start the iterations, designated in terms of s=1,2,..., we calculate the gradient vector for $\Delta f(\underset{\sim}{Y})$, denoted as $\underset{\sim}{G}_s$ at each iteration.

The gradient for the first iteration is derived by making use of the initial values obtained from the INVEST routine (see section A.2-3). Substituting the initial values into the ARMAV(m,n) model, we first estimate the determinant of the sum-of-squares-and-products matrix of $\underset{\sim}{a}_t$, $\underset{\sim}{A}_s$, for the s=1 iteration 1, where

$$\hat{a}_t = Y_t - \hat{\phi}_1 Y_{t-1} - \cdots - \hat{\phi}_m Y_{t-m} + a_t - \hat{\theta}_1 a_{t-1} - \cdots - \hat{\theta}_n a_{t-n} \qquad (A.2-28)$$

We then perturb each parameter individually by a magnitude of δ and compute the determinants of the resultant sum-of-squares-and-product matrix in the following s + 1 iteration, i.e., A_{s+1}.

Taking the differences of the determinants for two succeeding iterations yields the gradient.

$$g_i = \frac{A_s - A_{s+1}}{\partial} \qquad (A.2-29)$$

The above procedure is repeated for each parameter until the gradient g_i for every parameter shifted is obtained. The gradient vector G_s contains these elements.

(3) We next calculate the direction in which to move from the initial values according to

$$D_s = -H_s G_s \qquad (A.2-30)$$

(4) In order to move in a given direction D_s, we must also calculate the step size for each movement. This is accomplished by minimizing the function $f(Y_s + \lambda_s D_s)$ with respect to the step size λ. The choice of λ may be determined by minimizing the objective function of the ARMAV(m,n) process. Formally, the best step size is determined by minimizing

$$\lambda_s = \min_{\lambda} f(Y_s + \lambda D_s) \qquad (A.2-31)$$

which may be obtained by minimizing the objective function

$$\lambda_s = 2\,(\epsilon - |A_s|)\,|/D_s' G_s \qquad (A.2-32)$$

where ϵ is some arbitrarily chosen correction factor. This involves setting the initial value for λ and then successively decreasing its value by $\lambda_s = .5\lambda_s$

(5) Then we calculate

$$\delta_s = \lambda_s D_s \qquad (A.2-33)$$

and

$$\underset{\sim}{Y}_{s+1} = \underset{\sim}{Y}_s + \lambda_s \tag{A.2-34}$$

(6) We then calculate the second derivatives from differences of the gradients for two successive iterations:

$$\Delta \underset{\sim}{G}_s = \underset{\sim}{G}_{s+1} - \underset{\sim}{G}_s$$

(7) Using the $\Delta \underset{\sim}{G}_s$ and $\underset{\sim}{\delta}_s$ vectors computed above we next construct the A and B matrices

$$\underset{\sim}{A} = \frac{\underset{\sim}{\delta}_s \underset{\sim}{\delta}_s}{\underset{\sim}{G}_s \underset{\sim}{\delta}_s} \tag{A.2-35}$$

$$\underset{\sim}{B} = \frac{-\underset{\sim}{H}_s \Delta \underset{\sim}{G}_s \Delta \underset{\sim}{G}'_s \underset{\sim}{H}_s}{\Delta \underset{\sim}{G}'_s \underset{\sim}{H}_s \Delta \underset{\sim}{G}_s} \tag{A.2-36}$$

(8) Finally, we compute the Hessian matrix (note that H is an identity matrix before the first iteration) according to

$$\underset{\sim}{H}_{s+1} = \underset{\sim}{H}_s + \underset{\sim}{A} + \underset{\sim}{B} \tag{A.2-37}$$

(9) The above procedure is repeated for each succeeding iteration, during which the H matrix is successively approximated. Iteration continues until changes in the difference vector and the direction vector between two successive interations are less than or equal to some predetermined level, ϵ, for instance;

$$|\underset{\sim}{D}_s \cdot \Delta \underset{\sim}{G}_s| \leq c$$

On the other hand, if $|\underset{\sim}{D}_s \cdot \Delta \underset{\sim}{G}_s| > c$ we would return to step 2 and repeat the entire procedure replacing H_{s+1} for the previous $\underset{\sim}{H}_s$.

Since minimizing the differences between the $\Delta \underset{\sim}{G}_s$ and $\underset{\sim}{\delta}_s$ is equivalent to minimizing the objective function over a suitable choice of ϕ and Θ parameter values, the above procedure will lead to maximum likelihood estimations of the autoregressive-moving average vector model.

A.2.2.4 Minimization Criterion

The determinant of the sum-of-squares-and-products matrix of a_t, $A_0 = a_t' a_t$, where $a_t = (a_{1t}, a_{2t}, \ldots, a_{pt})$, the vector of residuals corresponding to equations 1 through p, is

$$D_A = \begin{vmatrix} \Sigma a_{1t}^2 & \Sigma a_{1t}a_{2t} & \cdots & \Sigma a_{1t}a_{pt} \\ \Sigma a_{2t}a_{1t} & \Sigma a_{2t}^2 & \cdots & \Sigma a_{2t}a_{pt} \\ \vdots & & \ddots & \vdots \\ \Sigma a_{pt}a_{1t} & \Sigma a_{pt}a_{2t} & \cdots & \Sigma a_{pt}^2 \end{vmatrix} \qquad \text{(A.2-38)}$$

which by denoting each entry as d_{ij} for $i=1,\ldots,p$ and $j=1,\ldots,p$ is

$$D_A = \begin{vmatrix} d_{11} & d_{12} & \cdots & d_{1p} \\ d_{21} & d_{22} & \cdots & d_{2p} \\ \vdots & & \ddots & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pp} \end{vmatrix} = d_{11} \cdot d_{22}^{(1)} \cdots d_{pp}^{(p-1)}$$

where

$$d_{22}^{(1)} = d_{22} - \frac{(d_{21})(d_{12})}{d_{11}}$$

$$\vdots$$

$$d_{pp}^{(p-1)} = d_{pp} - \frac{(d_{p,p-1})(d_{p-1,p})}{d_{p-1,p-1}}$$

In a two by two matrix (for ARV(1) model with p=2) the determinant of the squares and product matrix to be minimized is

$$D_A = \begin{vmatrix} \Sigma a_{1t}^2 & \Sigma a_{1t}a_{2t} \\ \Sigma a_{1t}a_{2t} & \Sigma a_{2t}^2 \end{vmatrix} \qquad \text{(A.2-39)}$$

$$= \Sigma a_{1t}^2 \, \Sigma a_{2t}^2 - \Sigma(a_{1t}a_{2t})^2$$

which must be positive if A_0 is positive definite, as $\Sigma a_{1t}^2 a_{2t}^2$ is always greater than $\Sigma(a_{1t}a_{2t})^2$. (Goldberger, 1964) Minimization is achieved by finding that set of estimates giving the smallest D_A.

A.2.2.5 Matrix Inversion and Characteristic Roots

Several subroutines are designed to compute the determinant and the inverse of a matrix and to compute the characteristic roots of the ARMAV(m,n) model in terms of the eigenvalues(and eigenvectors) of the parameter matrices. The matrix inversion routine is call"MATNV1" and the root finder is call "RTFIND". They are used in linear-least-squares estimation, initial value estimation (INVEST), and nonlinear estimation among others. The determinant of a matrix can be numerically estimated by an expansion:

$$|\underset{\sim}{A}| = a_{i1}C_{i1} + a_{i2}C_{i2} + \ldots + a_{ip}C_{ip} \qquad \text{(A.2-40)}$$

$$= \sum_{i=1}^{p} a_{ij}C_{ij}$$

where a_{ij} is the element of the matrix in the ith row jth column; C_{ij} is its corresponding cofactor which is defined by the relation

$$C_{ij} = (-1)^{i+j} |M_{ij}|$$

and $|M_{ij}|$ is the determinant of the minor of the cofactor. The minor is the submatrix that results if we eliminate the ith row and the jth column (intersecting at a_{ij}) of the matrix.

The inverse of the matrix accordingly is simply

$$\underset{\sim}{A}^{-1} = \underset{\sim}{C}'/|\underset{\sim}{A}| \qquad \text{(A.2-41)}$$

where $\underset{\sim}{C}'$ is the transpose of the cofactor $\underset{\sim}{C}$ of matrix $\underset{\sim}{A}$. For such an inverse to exist would require that $\underset{\sim}{A}$ be a square matrix.

The eigenvalues and eigenvectors to the $\underset{\sim}{A}$ matrix are solutions to the following system of homogeneous equations

$$(\underset{\sim}{A} - \lambda \underset{\sim}{I})\underset{\sim}{T} = 0 \qquad \text{(A.2-42)}$$

where λ is a lxp vector of eigenvalues and T is a pxp matrix of lxp eigenvectors, $T_1,\ldots,T_p$. Nontrivial solutions for the above equations exist if the determinant of the matrix of coefficients $(A - \lambda I)$ is equal to zero:

$$|A - \lambda I| = 0 \tag{A.2-43}$$

In general, the eigenvalues can be real and distinct, real but equal, or complex. Real roots define dynamics displaying the sum of exponential patterns of behavior; complex roots define cyclical patterns of behavior. For a two equation ARV(1) system, for example, the roots or eigenvalues are

$$\lambda_1, \lambda_2 = \frac{\phi_{111} - \phi_{221}}{2} \pm \frac{\sqrt{(\phi_{111} - \phi_{221})^2 - 4\,\phi_{221}\,\phi_{121}}}{2} \tag{A.2-44}$$

These roots are real if the "discriminant" under the square root is positive or equal to zero:

$$[(\phi_{111} - \phi_{221})^2 - 4\,\phi_{211}\,\phi_{121}] \geq 0$$

and they are complex if the discriminant is negative:

$$[(\phi_{111} - \phi_{221})^2 - 4\,\phi_{221}\,\phi 121] \leq 0$$

When roots are complex they form complex conjugate pairs such as

$$\begin{aligned} \lambda_1, \lambda_2 &= d\, e^{\pm i} \\ &= d\,(\cos\beta + i \sin\beta) \end{aligned} \tag{A.2-45}$$

where $i = \sqrt{-1}$, $d = \lambda_R^2 + \lambda_I^2$, $\beta = \tan^{-1}\left(\frac{\lambda I}{\lambda R}\right)$. The subscripts I and R stand for the real and the imaginary parts of roots into which complex roots may be split, for instance, the complex conjugates λ_1 and λ_2 may be written as

$$\lambda_1, \lambda_2 = \lambda_R \pm i\,\lambda_I \tag{A.2-46}$$

For further discussion see Li (1979, ch. 10).

A.2.3 Diagnostic Checking

Various diagnostic statistics are included in this system for testing the goodness of fit of the estimated ARMAV(m,n) model. First, inadequacy of the autoregressive component of the model is determined by examing the residual autocorrelations and partial autocorrelations for each **equation**. The autocorrelations are computed according to

$$r_a(k) = \frac{\sum_{t=1}^{n-k} (a_t - \bar{a}_t)(a_{t+k} - \bar{a}_{t+k})}{\sum_{t=1}^{n-k} (a_t - \bar{a}_t)^2} \qquad (A.2-47)$$

The partial autocorrelations are recursively computed from

$$\phi_{kk} = \frac{r_k - \sum_{p=1}^{k-1} \phi_{k-1,p}\, r_{k-p}}{1 - \sum_{p=1}^{k-1} \phi_{k-1,p}\, r_{k-p}} \qquad (A.2-48)$$

A two standard error confidence limit is defined for the above values:

$$r_a(k) \pm 2\sqrt{1/n} \qquad (A.2-49)$$

values exceeding this limit suggest inadequacy of the model.

Moreover, since it is known that the residual autocorrelations are inefficient estimates and tend to understate values at the lower order lags, an overall test of significance is also introduced (Box and Jenkins, 1970). The statistic is

$$Q = N \sum_{i=1}^{k} r_i^2 \qquad (A.2-50)$$

which is distributed as chi-square with k-m-n degrees of freedom, where k is the number of lagged correlations, m the order of the autoregressive component and n the order of the moving average component and N the number of observations.

Second, any needed revision in the interdependent relationships between variables or series will be revealed by examining the cross-correlations between the

residuals and the input series. The sample cross-correlations are computed according to

$$r_{ay}(k) = \frac{\sum_{k=0}^{n} (a_t - \bar{a}_t)(Y_{t+k} - \bar{Y}_{t+k})}{\left\{\sum_{k=0}^{n} (a_t - \bar{a}_t)^2 \sum_{k=0}^{n} (Y_{t+k} - Y_{t+k})^2\right\}^{\frac{1}{2}}} \qquad \text{(A.2-51)}$$

A two standard error confidence limit for the above is

$$r_{ay}(k) \pm 2 \sqrt{1/(n-k)} \qquad \text{(A.2-52)}$$

Values exceeding this limit will again suggest model inadequacy.

An overall test across all cross-correlations is also provided in the following statistic

$$S = N \sum_{k=1}^{p} r_{ay}(k) \qquad \text{(A.2-53)}$$

which is distributed as chi-squares with $k+1-(mp^2-1)$ degrees of freedom, where k is the number of lagged cross-correlations and (mp^2-1) is the total number of parameters in the autoregressive component of each equation in the system. Model inadequacy is suggested when the S statistic is larger than a certain critical chi-square value determined by the routine to be described in the ensuing section.

A.2.3.1 Chi-square Statistic

In order to determine the significance of the Q and S statistics we use a chi-square table to find the critical value up to 30 degrees of freedom, at 5% significance level, and for greater degrees of freedom we use a standard normal approximation of chi-squares:

$$\tfrac{1}{2} (Z_{\alpha} + \sqrt{2d-1})^2 \qquad \text{(A.2-54)}$$

where d is the degrees of freedom from the Q and S statistics and $\alpha = 1.645$ at 5% significance level. The search routine for determining the critical chi-square values is programmed in the CHISQ routine.

A.3.0 Forecasting

The Forecasting Routines employ the following algarithms.

A.3.1 PSI Weights (PSICOF)

$$(\underset{\sim}{I} - \underset{\sim}{\phi}_1 \underset{\sim}{B} - \underset{\sim}{\phi}_2 \underset{\sim}{B}^2 - \ldots - \underset{\sim}{\phi}_m \underset{\sim}{B}^m)(\underset{\sim}{\psi} + \underset{\sim}{\psi}\underset{\sim}{B} + \underset{\sim}{\psi}_2 \underset{\sim}{B}^2 + \ldots) = (\underset{\sim}{I} - \underset{\sim}{\theta}_1 \underset{\sim}{B} - \ldots - \underset{\sim}{\theta}_n \underset{\sim}{B}^n) \quad \text{(A.3-1)}$$

Equating like-terms in the above equality gives the following system of equations:

$$\underset{\sim}{\psi}_0 = \underset{\sim}{I}$$

$$\underset{\sim}{\psi}_1 = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_1$$

$$\underset{\sim}{\psi}_2 = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_1 + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_2$$

$$\underset{\sim}{\psi}_3 = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_2 + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_1 + \underset{\sim}{\phi}_3 \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_3 \qquad \text{(A.3-2)}$$

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$$\underset{\sim}{\psi}_{m-1} = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_{m-2} + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_{m-3} + \cdots + \underset{\sim}{\phi}_{m-1} \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_{n-1}$$

$$\underset{\sim}{\psi}_m = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_{m-1} + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_{m-2} + \cdots + \underset{\sim}{\phi}_m \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_n \qquad \text{, for } m=n$$

$$\underset{\sim}{\psi}_m = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_{m-1} + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_{m-2} + \cdots + \underset{\sim}{\phi}_m \underset{\sim}{\psi}_0 \qquad \text{, for } m>n$$

Setting the initial PSICOF matrix equal to identity, we may recursively compute higher order matrices, knowing in advance the estimated $\hat{\underset{\sim}{\phi}}$ matrices. For example, in an ARMAV(2,1) model where the estimates are $\underset{\sim}{\phi}_1$, $\underset{\sim}{\phi}_2$, and $\underset{\sim}{\theta}_1$, the PSICOF expansion is

$$\underset{\sim}{\psi}_0 = \underset{\sim}{I}$$

$$\underset{\sim}{\psi}_1 = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_0 - \underset{\sim}{\theta}_1$$

$$\underset{\sim}{\psi}_2 = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_1 + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_0$$

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$$\underset{\sim}{\psi}_m = \underset{\sim}{\phi}_1 \underset{\sim}{\psi}_{m-1} + \underset{\sim}{\phi}_2 \underset{\sim}{\psi}_{m-2}$$

So, for

$$\hat{\underset{\sim}{\phi}}_1 = \begin{bmatrix} .45 & .65 \\ .20 & .50 \end{bmatrix} \qquad \hat{\underset{\sim}{\phi}}_2 = \begin{bmatrix} .30 & .15 \\ .25 & .28 \end{bmatrix} \qquad \hat{\underset{\sim}{\Theta}}_1 = \begin{bmatrix} .25 & .89 \\ .15 & .39 \end{bmatrix}$$

the PSICOF are

$$\hat{\underset{\sim}{\psi}}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{\underset{\sim}{\psi}}_1 = \begin{bmatrix} .45 & .65 \\ .20 & .50 \end{bmatrix} - \begin{bmatrix} .25 & .89 \\ .15 & .39 \end{bmatrix} = \begin{bmatrix} .20 & -.24 \\ .05 & .11 \end{bmatrix}$$

$$\hat{\underset{\sim}{\psi}}_2 = \begin{bmatrix} .45 & .65 \\ .20 & .50 \end{bmatrix} \times \begin{bmatrix} .20 & -.24 \\ .05 & .11 \end{bmatrix} + \begin{bmatrix} .30 & .15 \\ .25 & .28 \end{bmatrix} = \begin{bmatrix} .42 & .11 \\ .32 & .29 \end{bmatrix}$$

$$\hat{\underset{\sim}{\psi}}_3 = \begin{bmatrix} .45 & .65 \\ .20 & .50 \end{bmatrix} \times \begin{bmatrix} .42 & .11 \\ .32 & .29 \end{bmatrix} + \begin{bmatrix} .30 & .15 \\ .25 & .28 \end{bmatrix} \times \begin{bmatrix} .20 & -.24 \\ .05 & .11 \end{bmatrix} = \begin{bmatrix} .43 & .18 \\ .31 & .14 \end{bmatrix}$$

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These matrices are used in several parts of this software. Specifically, they are necessary in computing (1) the forecast confidence intervals, and (2) the adaptive forecasting routine.

A.3.2 Forecasting Algorithm (FORCST)

The forecasting algorithm incorporates two major features. First, it permits forecasting with leading indicators using multiple series. The forecasts are generated either from the full vector model or from a restricted single equation model. Second, it accommodates regular and seasonal difference operations to restore stationarity to the original series. The difference operators are explicitly incorporated in the forecasting functions.

Mathematically, our problem is one of expanding a forecasting system of the following form from an ARMAV(m,n) model.

$$\begin{bmatrix} (1-B^{s_2})^{d_{11}}(1-B)^{d_{12}} & Y_{1t} \\ (1-B^{s_2})^{d_{21}}(1-B)^{d_{22}} & Y_{2t} \\ \vdots & \vdots \\ (1-B^{s_p})^{d_{21}}(1-B)^{d_{22}} & Y_{pt} \end{bmatrix} \begin{bmatrix} \phi_{111} & \phi_{121} & \cdots & \phi_{1p1} \\ \phi_{211} & \phi_{221} & \cdots & \phi_{2p1} \\ \vdots & & \ddots & \\ \phi_{p11} & \phi_{p21} & & \phi_{pp1} \end{bmatrix} \begin{bmatrix} (1-B^{s_1})^{d_{11}}(1-B)^{d_{12}} Y_{1t-1} \\ (1-B^{s_2})^{d_{21}}(1-B)^{d_{22}} Y_{2t-1} \\ \vdots \\ (1-B^{s_p})^{d_{p1}}(1-B)^{d_{p2}} Y_{pt-1} \end{bmatrix} + \cdots +$$

$$\begin{bmatrix} a_{1t} \\ a_{2t} \\ \vdots \\ a_{pt} \end{bmatrix} - \begin{bmatrix} \theta_{111} & \theta_{121} & \cdots & \theta_{1p1} \\ \theta_{121} & \theta_{221} & & \theta_{2p1} \\ \vdots & & \ddots & \\ \theta_{p11} & \theta_{p21} & \cdots & \theta_{pp1} \end{bmatrix} \begin{bmatrix} a_{1t-1} \\ a_{2t-1} \\ \vdots \\ a_{pt-1} \end{bmatrix} - \cdots -$$

$$\begin{bmatrix} \theta_{11n} & \theta_{12n} & \cdots & \theta_{1pn} \\ \theta_{21n} & \theta_{22n} & & \theta_{2pn} \\ \vdots & & \ddots & \\ \theta_{p1n} & \theta_{p2n} & & \theta_{ppn} \end{bmatrix} \begin{bmatrix} a_{1t-n} \\ a_{2t-n} \\ \vdots \\ a_{pt-n} \end{bmatrix} \qquad \text{(A.3-3)}$$

The general system forecasting equation is:

$p =$ total number of series used in forecast $1 \leq p \leq 5$

ϕ_{ijk} = autoregressive parameter of the k^{th} matrix, in row i, col j

θ_{ijk} = moving average parameter of the k^{th} matrix, in row i, col j

B = backspace operator: $BY_t = Y_{t-k}$

S_i = seasonal operator of indicator i

d_{i1} = number of seasonal differences

d_{i2} = number of regular differences

α_{ij} = lag of series j with respect to series i (L_{ij} = 0)

Y_{it} = observation of i^{th} indicator at time t

a_{it} = residual of i^{th} indicator at time t

Disregarding the MA terms, this system is equivalent to:

$$0 = \begin{bmatrix} (1-B^{s_1})^{d_{11}}(1-B)^{d_{12}} Y_{1t} \\ \cdot \\ \cdot \\ \cdot \\ (1-B^{s_p})^{d_{p1}}(1-B)^{d_{p2}} Y_{pt} \end{bmatrix} - \begin{bmatrix} \phi_{111} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{p11} \end{bmatrix} (1-B^{s_1})^{d_{11}}(1-B)^{d_{12}} Y_{1t-1} - \dots p$$

$$- \begin{bmatrix} \phi_{11n} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{p1n} \end{bmatrix} (1-B^{s_1})^{d_{11}}(1-B)^{d_{12}} Y_{1t-n}$$

$$- \begin{bmatrix} \phi_{1p1} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{pp1} \end{bmatrix} (1-B^{s_p})^{d_{p1}}(1-B)^{d_{p2}} Y_{pt-1} - \dots -$$

$$- \begin{bmatrix} \phi_{1pn} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{ppn} \end{bmatrix} (1-B^{s_p})^{d_{p1}}(1-B)^{d_{p2}} Y_{pt-n}$$

Using the matrix column portion $\underset{\sim}{\phi}_i = [\underset{\sim}{\phi}_{1i} \mid \underset{\sim}{\phi}_{2i} \mid \dots \mid \underset{\sim}{\phi}_{pi}]$, calling $\hat{Y}_{it} = (1 - B^{si})^{d_{i1}} (1-B)^{d_{i2}} Y_{it}$ and letting $\underset{\sim}{I}_{pxp} = [\underset{\sim}{I}_{1i} \mid I_{2i} \mid \dots \mid I_p]$

$$\begin{aligned} 0 = \; & \underset{\sim}{I}_1 \hat{\underset{\sim}{Y}}_{1t} - \phi_{11}\hat{Y}_{1t-1} - \phi_{12}\hat{Y}_{1t-2} + \dots + \phi_{1n}\hat{Y}_{1t-n} \\ & + I_2 \hat{Y}_{2t} - \phi_{21}\hat{Y}_{2t-1} - \phi_{22}\hat{Y}_{2t-2} + \dots + \phi_{2n}\hat{Y}_{2t-n} \\ & + I_p \hat{Y}_{pt} - \phi_{p1}\hat{Y}_{pt-1} - \phi_{p2}\hat{Y}_{pt-2} - \dots - \phi_{pn}\hat{Y}_{pt-n} \end{aligned} \tag{A.3-4}$$

Note that this expression of the system breaks the system into p equations, where each equation i depends only upon the i column of each coefficient matrix and the ith indicator. The differencing within each indicator can then be expressed by expanding the vectors ϕ_{pi} in the manner indicated by the expression $(1-B^{s_i})^{d_{i1}} (1-B)^{d_{i2}}$.

Example

$$p = 2 \quad n = 2 \quad s_1 = 2 \quad d_{11} = 1 \quad d_{12} = 1$$

$$s_2 = 3 \quad d_{21} = 1 \quad d_{22} = 1$$

$$\begin{aligned} \hat{Y}_{1t} &= (1-B^2)(1-B)\, Y_{1t} = (1 - B - B^2 + B^3)\, Y_{1t} \\ &= Y_{1t} - Y_{1t-1} - Y_{1t-2} + Y_{1t-3} \end{aligned}$$

$$\begin{aligned} \hat{Y}_{2t} &= (1-B^3)(1-B)\, Y_{2t} = (1 - B - B^3 + B^4)\, Y_{2t} \\ &= Y_{2t} - Y_{2t-1} - Y_{3t-3} + Y_{2t-4} \end{aligned}$$

Substituting into expression (A.3-4) the above expression becomes

$$0 = \begin{bmatrix} Y_{1t} \\ 0 \end{bmatrix} - \begin{bmatrix} \phi_{111} \\ \phi_{211} \end{bmatrix} Y_{1t-1} - \begin{bmatrix} \phi_{112} \\ \phi_{212} \end{bmatrix} Y_{1t-2}$$

$$- \begin{bmatrix} 0 \\ Y_{2t} \end{bmatrix} - \begin{bmatrix} \phi_{121} \\ \phi_{221} \end{bmatrix} Y_{2t-1} - \begin{bmatrix} \phi_{122} \\ \phi_{222} \end{bmatrix} Y_{2t-2}$$

$$= \begin{bmatrix} Y_{1t} - Y_{1t-1} - Y_{1t-2} + Y_{1t-3} \\ 0 \end{bmatrix} - \begin{bmatrix} \phi_{111}(Y_{1t-1} - Y_{1t-2} - Y_{1t-3} + Y_{1t-4}) \\ \phi_{211}(Y_{1t-1} - Y_{1t-2} - Y_{1t-3} + Y_{1t-4}) \end{bmatrix}$$

$$- \begin{bmatrix} \phi_{112}\ (Y_{1t-2} - Y_{1t-3} - Y_{1t-4} + Y_{1t-5}) \\ \phi_{212}\ (Y_{1t-2} - Y_{1t-3} - Y_{1t-4} + Y_{1t-5}) \end{bmatrix} + \begin{bmatrix} 0 \\ Y_{2t} - Y_{2t-1} - Y_{2t-3} + Y_{2t-4} \end{bmatrix} -$$

$$\begin{bmatrix} \phi_{121}\ (Y_{2t-1} - Y_{2t-2} - Y_{2t-4} + Y_{2t-5}) \\ \phi_{221}\ (Y_{2t-1} - Y_{2t-2} - Y_{2t-4} + Y_{2t-5}) \end{bmatrix} - \begin{bmatrix} \phi_{122}\ (Y_{2t-2} - Y_{2t-3} - Y_{2t-5} + Y_{2t-6}) \\ \phi_{222}\ (Y_{2t-2} - Y_{2t-3} - Y_{2t-5} + Y_{2t-6}) \end{bmatrix}$$

Collecting like terms of Y_{1t}, Y_{1t-1}, etc we get

$$0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} Y_{1t} + \begin{bmatrix} -1 - \phi_{111} \\ - \phi_{211} \end{bmatrix} Y_{1t-1} + \begin{bmatrix} -1 + \phi_{111} - \phi_{112} \\ \phi_{211} \quad \phi_{212} \end{bmatrix} Y_{1t-2} + \begin{bmatrix} 1 + \phi_{111} + \phi_{112} \\ \phi_{211} + \phi_{212} \end{bmatrix} Y_{1t-3}$$

$$+ \begin{bmatrix} -\phi_{111} + \phi_{112} \\ \phi_{211} + \phi_{212} \end{bmatrix} Y_{1t-4} + \begin{bmatrix} -\phi_{112} \\ \phi_{212} \end{bmatrix} Y_{1t-5}$$

$$+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} Y_{2t} + \begin{bmatrix} -\phi_{121} \\ -1 - \phi_{221} \end{bmatrix} Y_{2t-1} + \begin{bmatrix} \phi_{121} - \phi_{122} \\ +\phi_{221} - \phi_{222} \end{bmatrix} Y_{2t-2} + \begin{bmatrix} \phi_{122} \\ -1 + \phi_{222} \end{bmatrix} Y_{2t-3}$$

$$+ \begin{bmatrix} \phi_{121} \\ 1 + \phi_{221} \end{bmatrix} Y_{2t-4} + \begin{bmatrix} -\phi_{121} + \phi_{122} \\ -\phi_{221} + \phi_{222} \end{bmatrix} Y_{2t-5} + \begin{bmatrix} -\phi_{122} \\ -\phi_{222} \end{bmatrix} Y_{2t-6}$$

Collecting these terms into vectors of the form $\begin{bmatrix} Y_{1t-j} \\ Y_{2t-j} \end{bmatrix}$, $j = 0, \ldots, n$ and solving for $\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix}$ we get

$$\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = - \begin{bmatrix} -1 - \phi_{111} & - 121 \\ -\phi_{211} & -1 - \phi_{221} \end{bmatrix} \begin{bmatrix} Y_{1t-1} \\ Y_{2t-1} \end{bmatrix} - \begin{bmatrix} -1 + \phi_{111} - \phi_{112} & \phi_{121} - \phi_{122} \\ \phi_{211} - \phi_{212} & \phi_{221} - \phi_{222} \end{bmatrix} \begin{bmatrix} Y_{1t-2} \\ Y_{2t-2} \end{bmatrix}$$

$$- \begin{bmatrix} 1+\phi_{111}-\phi_{112} & \phi_{121} \\ \phi_{211}+\phi_{212} & -1+\phi_{222} \end{bmatrix} \begin{bmatrix} Y_{1t-3} \\ Y_{2t-3} \end{bmatrix} - \begin{bmatrix} -\phi_{111}+\phi_{112} & \phi_{121} \\ -\phi_{211}+\phi_{212} & 1+\phi_{221} \end{bmatrix} \begin{bmatrix} Y_{1t-4} \\ Y_{2t-4} \end{bmatrix}$$

$$- \begin{bmatrix} -\phi_{112} & -\phi_{121}+\phi_{122} \\ -\phi_{212} & -\phi_{221}+\phi_{222} \end{bmatrix} \begin{bmatrix} Y_{1t-5} \\ Y_{2t-5} \end{bmatrix} - \begin{bmatrix} 0 & -\phi_{122} \\ 0 & -\phi_{222} \end{bmatrix} \begin{bmatrix} Y_{2t-6} \\ Y_{2t-6} \end{bmatrix}$$

A.3.3 Forecast Confidence Intervals (FRCONF)

A two standard error confidence interval is estimated for each forecast. The confidence interval is:

$$\underset{\sim}{Y}_t(\ell) \pm 2\sqrt{D_{iit}(\ell)} \qquad \text{(A.3-5)}$$

where $D_{iit}(\ell)$ is the ith diagonal element of the variance-covariance matrix of forecast errors for ℓ-step-ahead forecasts:

$$\underset{\sim}{D}_t(\ell) = \underset{\sim}{\sigma}_a^2 + \underset{\sim}{\pi}_1 \underset{\sim}{\sigma}_a^2 \underset{\sim}{\pi}_1' + \dots + \underset{\sim}{\pi}_{\ell-1} \underset{\sim}{\sigma}_a^2 \underset{\sim}{\pi}_{\ell-1}' \qquad \text{(A.3-6)}$$

The $\underset{\sim}{\pi}_i$'s are the PSICOF (see preceeding section) and $\underset{\sim}{\sigma}_a^2$ can be approximated by the sample residual sum-of-squares-and-products matrix,

$$\underset{\sim}{\sigma}_a^2 = \frac{1}{n} \begin{bmatrix} \Sigma a_{1t}^2 & \Sigma a_{1t}a_{2t} & \dots & \Sigma a_{1t}a_{pt} \\ \Sigma a_{2t}a_{1t} & \Sigma a_{2t}^2 & & \Sigma a_{2t}a_{pt} \\ \vdots & & \ddots & \\ \Sigma a_{pt}a_{1t} & \Sigma a_{pt}a_{2t} & \dots & \Sigma a_{pt}^2 \end{bmatrix} \qquad \text{(A.3-7)}$$

A.3.4 Forecast Updating (FRUPDT)

In many operational settings in which forecasts are made on a continuous basis, it is expedient to update forecasts from time to time with the latest piece of information without recomputing the previous forecasts. This can be readily done for an autoregressive-moving average vector model, requiring only the previous forecasts and the new updating information. The method is known as adaptive forecasting.

Adaptive forecasting seeks to improve the accuracy of present forecasts by means of the current one-step-ahead forecast error computed from the latest updating information. Assuming that we have a series of ℓ-step-ahead forecasts, generated from time t, and a new piece of data becomes available at time t+1; our one-step-ahead forecast error would be $Y_{t+1} - Y_t(1)$, the difference between the new data from time t+1 and the one-step-ahead forecast for time t+1 from time t. By properly weighting this measure of forecast inaccuracy and adding the result to the one-step-ahead forecasts we may obtain significant improvement over the previous forecasts (Box and Jenkins, 1970). Thus, our adaptive forecasting algorithm for our vector model is

$$\underset{\sim}{Y}_{t+1}(\ell) = \underset{\sim}{Y}_t(\ell + 1) + \underset{\sim}{\psi}_\ell[\underset{\sim}{Y}_{t+1} - \underset{\sim}{Y}_t(1)]$$

where $Y_{t+1}(\ell)$ is a pxp matrix of updated forecasts for time t+1; $\underset{\sim}{Y}_t(\ell + 1)$ is a pxℓ + 1 matrix of previous forecasts generated at time t; $\underset{\sim}{Y}_{t+1} - Y_t(1)$ is a px1 vector of one-step-ahead forecast errors; ψ_ℓ is the PSICOF matrix for the ℓth step ahead forecast, containing nonzero diagonal elements and zero off-diagonal elements.

The practicality of this method stems from the fact that in order to make use of the method we need only to keep track of our previous forecasts and of our one-step-ahead forecast errors, albeit we might periodically want to reexamine our forecasting models with the original data. In short, this method permits us to learn from past errors in improving our present forecasts.

A.3.5 Z Statistic and Standard Normal Probability Function (PRBZ)

A standard normal "Z" statistic is computed for each forecast. It provides a standardized measure of the deviation of a random variable from its mean. Taking the mean of an indicator or a series to imply its norm, the Z statistic may be construed as an indication of the degree of abnormality, or deviation from norm, in our forecasts. Assuming stationarity for our series, we may expect the mean

and the variance of our series to remain constant over time. Thus, even though the mean and the variance of the future values that we are attempting to forecast are unknown, we may use the existing mean and variance as approximations. Accordingly, the Z value corresponding to a forecast may be computed from

$$Z_\ell = \frac{Y_t(\ell) - \bar{Y}_t}{S_y} \qquad \text{(A.3-9)}$$

where $\bar{Y}_t$ and S_y are the sample mean and sample standard deviation of the existing series, and $Y_t(\ell)$ is the ℓth-step-ahead forecast generated from time origin t.

Statistically, Z is known to be normally distributed with mean zero and variance one. Its probability density function is

$$f(Z) = \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} \qquad \text{(A.3-10)}$$

Integrating this function from 0 to Z would give us the probability for the region between the mean and the given Z_ℓ value. The probability that a deviation from the mean will exceed Z_ℓ is given by subtracting the integral for the region between 0 and Z from .5; that is

$$P(Z > Z_\ell) = .5 - [\int_0^Z \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ] \qquad \text{(A.3-11)}$$

In the CEWM forecaster, the computation of the Z value is performed in the main program and of the probability estimate is computed in the PRBZ routine. Subroutine PRBZ performs the following computation.

The cumulative density function

$$F(\chi) = \frac{1}{\sqrt{2\pi}} \int_0^\chi e^{-\frac{1}{2}y^2} dy \qquad \text{(A.3-12)}$$

is, in general, difficult to evaluate for an arbitrary value χ. The approach used in this system is to estimate the value of $F(\chi)$ by creating an interpolating polynomial. By picking points (χ_i, Y_i), i = 0, n and $Y_i = F(\chi_i)$, and fitting a polynomial $P_n(\chi)$ through these points, such that $P_n(\chi_i) = F(\chi_i)$ a good estimate

of the $F(\chi)$ function can be obtained:

$$P_n(\chi) = P_{n-1}(\chi) + f[\chi_0, \chi_1, \cdots, \chi_n] \prod_{j=0}^{n-1} (\chi - \chi_j)$$

$$\text{where } f[\chi_0, \cdots, \chi_n] = \frac{f[\chi_0, \cdots, \chi_{n-1}] - f[\chi_1, \cdots, \chi_n]}{\chi_0 - \chi_n} \qquad \text{(A.3-13)}$$

and $f[\chi_0] = f(\chi_0)$

Using the data points in the table below (F (x) values are from the 21st edition of the CRC Standard Mathematical Tables):

i	χ_i	$Y_i = f(\chi_i)$
0	0	.5
1	1	.8413
2	2	.9772
3	3	.9987
4	4	1.000
5	.5	.6915
6	2.5	.9938

$P_6(\chi)$ was found to be

$$P_6(\chi) = P_5(\chi) + .00050608\ (\chi)\ (\chi-1)\ (\chi-2)(\chi-3)(\chi-4)(\chi-5)$$

$$= P_5(\chi) + .00050608\ \chi(\chi^5 - 10.5\chi^4 + 40\chi^3 - 67.5\chi^2 + 49\chi-12)$$

$$= .5 + .390295221\chi+.041201314\chi^2 - .137276229\chi^3$$

$$+ .055993667\chi^4 - .009519051\chi^5 + .00060508\chi 6$$

Empirical comparisons have shown that this polynomial has a maximum error of approximately .001, which was considered to be sufficiently accurate for our purposes.

A.3.6 Combined Forecasts (COMBFR)

When several alternative forecasting methods are available it is sometimes useful to combine their results in order to complement their strength and weaknesses. The forecasts to be combined must come from the same series though they are generated by different forecasting methods. Thus, the method is intended to compensate for the differences between methods, not between forecast indicators.

The method we use, as originally developed by Granger and Newbold (1969, 1974), permits us to combine multiple frequency forecasts. The combined forecast is a linear combination of the original forecasts:

$$\underset{\sim}{C}_t = \underset{\sim}{K}_t \underset{\sim}{Y}_t(\ell) \tag{A.3-14}$$

where, $\underset{\sim}{K}_t = (k_{1t}, k_{2t}, \ldots, k_{nt})$, is the combine forecast weights, and $Y_t(\ell)$ is a matrix of alternative forecast series,

$$\underset{\sim}{Y}_t(\ell) = [\underset{\sim}{Y}_{1t}(\ell), \underset{\sim}{Y}_{2t}(\ell), \ldots, \underset{\sim}{Y}_{nt}(\ell)].$$

Granger and Newbold (1974) suggested that the forecast weights may be computed according to the following relation

$$k_{it} = \{ \sum_{t=T-v}^{T-1} e_{it}^2 \}^{-1} / \{ \sum_{i=1}^{m} (\sum_{t=T-v}^{T-1} e_{it}^2)^{-1} \} \qquad \text{for } i=1,\ldots,m \tag{A.3-15}$$

where $e_{it} = Y_{it-\ell} - Y_{it}(\ell)$, which is the forecast errors for ℓ-step-ahead forecasts. It is suggested that the forecast error for the last twelve periods be used to compute k_{it}, which means in the above equation we set v=12. Our decision rule accordingly specifies that if the number of our series from which we generate the forecasts has more than 90 observations we set v=12; otherwise we set v=.12n, where n is the number of total observations in the time series.

APPENDIX B

Data Coding Procedure

B.0 Data Coding Rules

The crisis indicators are developed from data on 17 biographical attributes. They include:

- name
- date of death
- date of birth
- place of birth (native place)
- generation
- field army affiliation by generation
- commander or commissar by generation
- date entered party
- military or civilian
- combat experience
- awards
- civilian education
- military education
- military-region affiliation by generation
- functional affiliation by generation
- position level or rank by year
- province or military-region affiliation, by year.

These attributes are selected according to criteria pertinent to data availability, institutional loyalty, and interest group behavior.

Data are constructed on an annual basis from 1974 to 1978 and are intended as extension of an existing data set developed at Rand (Sung, R-1665-ARPA) for the period 1957 to 1973. The coding rules and procedures of the preceding study are used in developing the present data set. The result is a set of data organized in a machine readable form and coded on computer cards.

Biographical information for each elite is coded on three cards. Card one contains information pertaining to name, dates of birth and death, place of birth, generation, field-army affiliation by generation, commander or commissar by generation and by year, military region by generation and by year, functional affiliation by generation and by year, civilian or military background, and province/military region by year. Card two contains information pertaining to military, party, and governmental positions by year, and promotion and demotion (or both) frequencies by year. Card three, which pertains only to new elites included in our sample since 1974, contains information pertaining to year of birth, place of birth, generation, combat experience, award, civilian and military education. These background materials are available for the pre-1974 sample of elites in the Rand data set (Sung, R-1665-ARPA) and are not reproduced here. Table 6 displays the coding data layout for the computer cards. The coding rules used to register data numerically on these cards are described below.

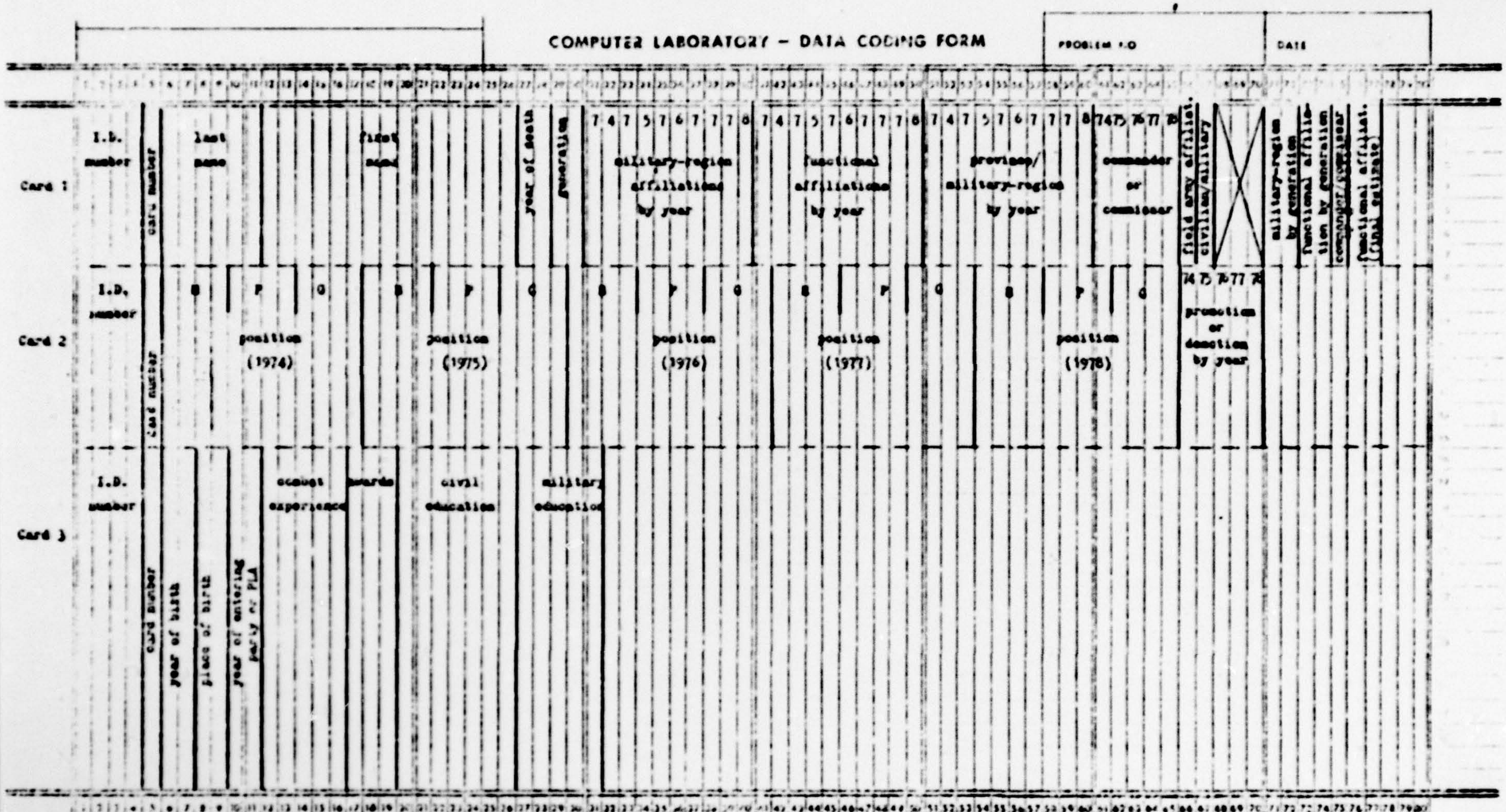
Card 1

1. Name
2. Date of death
3. Date of birth
4. Place of birth (native place)

   1 = Anhwei, 2 = Chekiang, 3 = Fukien, 4 = Heilungkiang, 5 = Honan, 6 = Hopeh, 7 = Hunan, 8 = Hupeh, 9 = Inner Mongolia, 10 = Kansu, 11 = Kiangsi, 12 = Kiangsu, 13 = Kirin, 14 = Kwangsi, 15 = Kwangtung, 16 = Kweichow, 17 = Liaoning, 18 = Ninghsia, 19 = Peking, 20 = Shanghai, 21 = Shansi, 22 = Shantung, 23 = Shensi, 24 = Sinkiang, 25 = Szechwan, 26 = Tibet, 27 = Tientsin, 28 = Tsinghai, 29 = Yunnan, 30 = Taiwan

Table 6

Data Layout on Computer Cards



M = military; P = party; G = government

5. Generation*

GI = Pre-May 1928, G2 = June 1928-November 1931, G3 = December 1931-December 1936, G4 = January 1937-December 1940, G5 = January 1941-August 1945, G6 = September 1945-October 1950, G7 = November 1950-September 1954, G8 = October 1954-September 1959, G9 = October 1959 -December 1963, G10 = January 1964-January 1967, G11 = February 1967-March 1969, G12 = April 1969-September 1973, G13 = October 1974-

-September 1977, FE = Final Estimate

6. Field-army (FA) affiliation by generation**

1. = 1st FA, 2 = 2d FA, 3 = 3d FA, 4 = 4th FA, 5 = North China FA, China FA, 6 = the Center

7. Commander (cdr) or commissar (csr) by generation***
1 = commissar, 2 = commander, 3 = both

8. Military or civilian
1 = civilian, 2 = military

---

*Generation affiliation is determined by the date of entry into the Army for military personnel, or into the CCP for civilians. In addition to the twelve generations identified in the Rand study we have added a thirteenth. The period of each of the generations represents more or less a cycle of crisis in the history of the CCP and the Red Army. Thus generation thirteenth is identified by the period bounded by the dates of the 10th and the 11th party congresses, roughly October, 1974 to September, 1977.

**Field-army affiliation, commander or commissar, military-region affiliation, and functional affiliation are data coded according to the generation period in which they occurred.

***Commander/commissar categories apply to military district, garrison, military regions, and service arms. In addition, the Director and Deputy Director of General Staff Department are classified as commander, whereas the Director and Deputy Director of General Political Department are classified as commissar.

9. Military-region affiliation by generation

1 = Sinkiang, 2 = Kunming, 3 = Nanking, 4 = Canton,
5 = Peking, 6 = Ch'engtu, 7 = Fuchou, 8 = Lanchou,
9 = Shenyang, 10 = Tsinan, 11 = Wuhan, 12 = Center

10. Functional affiliation by generation*

1 = Armor, 2 = Artillery, 3 = Engineers, 4 = Infantry, 5 = Signal Corps, 6 = Railway Corps, 7 = General Political Department, 8 = 2d Artillery (Missiles), 9 = Public Security Force, 10 = Air Force, 11 = Navy, 12 = General Chief of Staff, 13 = General Rear Service, 14 = Propaganda and Education (including ministries: Culture, Education, Higher Education, Internal Affairs Public Health), 15 = Industry and Communication (Building Construction, Building Materials, Chemical Industry, 1st, 2d, 3d, 4th, 5th, 6th, 7th, and 8th Machinebuilding, Fuel & Chemical Industry, Communication, Geology, Light Industry, Textile Metallurgical Industry, Petroleum

Industry, Post and Telecommunication, Railway, Water Conservation, Allocation of Materials), 16 = Finance and Trade (Commerce, Finance, Food, Foreign Trade, Economic with Foreign Countries, Planning Commission), 17 = Foreign Affairs (Foreign Affairs), 18 = Agriculture and Forestry (Agriculture, Forestry, Aquatic Products, Labor, State Farm and Land Reclamation), 19 = Political, Legal, Organization and Personnel

11. Province or military-region affiliation by year**

1 = Anhwei, 2 = Chekiang, 3 = Fukien, 4 = Heilungkiang, 5 = Honan, 6 = Hopeh, 7 = Hunan, 8 = Hupeh, 9 = Inner Mongolia, 10 = Kansu, 11 = Kiangsi, 12 = Kiangsu, 13 = Kirin, 14 = Kwangsi, 15 = Kwangtung, 16 = Kweichow, 17 = Liaoning, 18 = Ninghsia, 19 = Peking, 20 = Shanghai, 21 = Shansi, 22 = Shantung, 23 = Shensi, 24 = Sinkiang, 25 = Szechwan, 26 = Tibet, 27 = Tientsin, 28 = Tsinghai, 29 = Yunnan, 30 = Center MR, 31 = Sinkiang MR (including 24), 32 = Kunming MR (including 16, 29), 33 = Nanking MR (including 1,2,11,20), 34 = Canton MR (including 16,15,7), 35 = Peking MR (including 6,9, 21,19), 36 = Ch'engtu MR (including 25,26), 37 = Fuchou MR (including 3,11), 38 = Lanchou MR (including 10,18,23,28), 39 = Shenyang MR (including 4,13,17), 40 = Tsinan MR (including 22,27), 41 = Wuhan MR (including 5,8).

---

*As distinct from the Rand study we collapse political-legal and organizational-personnel backgrounds into category 19, instead of treating them separately. We also include in 19 vice premier, the minister and vice minister of National Defense Ministry, the Scientific and Technological commissar of the National Defense Ministry and the chief of the military academy. Moreover, categories 1 through 13 pertain national level People's Liberation Army organizations; categories 14 through 18 pertain to national level party and governmental organizations; category 19 also contains subnational level party and governmental commissions.

**Provincial affiliations are coded at the lowest level for which data are available. The levels, considered in ascending order, are: province, military district, region, and center (the Peking district).

Card 2

12. Position level or rank, by year (1956-1973)*

Military (M)

4 = National level

41 = Chairman, Military Affairs Committee (MAC)
42 = Vice Chairman, Military Affairs Committee
43 = Defense Minister
44 = Vice Defense Minister
45 = Chief of service arm
46 = Deputy Chief of service arm
47 = Political Commissar (PC) of service arm
48 = Deputy Political Commissar of service arm
49 = Unidentified position

3 = Regional level

31 = Commander of military region
32 = Political Commissar of military region
33 = Deputy Commander of military region
34 = Deputy Political Commissar of military region
35 = Chief of Staff of military region
30 = Unidentified position

2 = District level

21 = Commander of military district
22 = Political Commissar of military district
23 = Deputy Commander of military district
24 = Deputy Political Commissar of military district

Party (P)

4 = National level

41 = Politburo member
42 = Politburo alternate member
43 = Full member of Central Committee
44 = Alternate member of Central Committee

---

*Position is identified at end of each year according to military region, functional affiliations, commander/commissar distrinction, promotion/demotion/both categories. When no position is observed, no affiliation of any sort is assumed. In addition, we include under unidentified position: (1) purged or demoted cases, (2) uncodable position, (3) missing data, (4) post-purge year. Uncodable position also includes cosmetic or nominal position to which no actual political authority is attached.

3 = Regional level

31 = 1st Secretary
32 = Secretary

2 = Provincial level

21 = 1st Secretary
22 = 2d Secretary
23 = Secretary
24 = Deputy Secretary
25 = Standing member

Government (G)

4 = National level

41 = Chief and Deputy Chief of State
42 = Premier
43 = Vice Premier
44 = Minister
45 = Vice Minister
46 = Director

2 = Provincial level

21 = Governor (Chairman, Revolutionary Committee)
22 = Vice Governor (Vice Chairman, Revolutuionary Committee)
23 = Standing member, *Revoluationary Committee*

98 = Dismissed and purged

99 = Disappeared and attacked

13. Promotion and Demotion*

*Promotion and demotion are coded according to movement between two successive years for the following categories: position t_1 and position t_2, military region t_1 and military region t_2, functional affiliations t_1 and functional affiliations t_2 province t_1 and province t_2, commander/commissar t_1 and commander/commissar t_2.

Card 3

16. Year of birth

17. Place of birth

18. Year of entering party

19. Combat experience

    1 = Korea (1950-1953), 2 = Taiwan Straits (1955, 1958, 1962), 3 = Sino-Indian border (1962), 4 = North Vietnam (1965-1971), 5 = Sino-Soviet border (1965-)

20. Awards

    1 = August 1 medal, 2 = Independent Liberty medal, 3 = Liberation medal

21. Civilian education

    1 = grade school, 2 = high school, 3 = college, 4 = study in USSR, 5 = study in Germany, 6 = study in Japan, 7 = study in France

22. Military education

    1 = basic school, 2 = anti-Japanese college, 3 = Nanking Staff College, 4 = Peking War College, 5 = Soviet advisers (including study in Soviet military schools)

According to these categories we group the elites into factions. The total number of elites in each category represents the base score from which the crisis indicator is constructed. Assuming that a large number of demotions signifies abnormal strain on the political system and thus the occurence of crisis, we count the total frequency of demotions in each elite faction. Comparing the demotion frequencies for competing factions gives us a further measure of inter-factional conflict.

Demotion is taken here to mean not only movement from a higher position to a lower one but also purge, dismissal, and disappearance. To compute demotion we compare the lowest position held by an individual for two consecutive years. If the position held in the earlier year is higher than that of the latter than a demotion is assumed to have occurred. (In contrast,

a movement in the reverse direction signifies a promotion). Demotion frequencies are counted for elite factions, within the party, the governmental and the military hierarchies. A specific number of such positions have been charted in the previous Rand study (Sung, R-1665-ARPA) through which demotion or promotion patterns may be traced. As the procedure is clearly described in the preceding study no further attempt will be made here to explicate the counting rules. All further explanations are given in the previous study.

The policy indicators are developed from aggregate economic data. With the exception of the agricultural and industrial output measures most data are estimated in constant dollars. These include Gross National Product, import, export and defense expenditures. Since annual agricultural and industrial output measures are difficult to obtain and not available for many years, only estimates of performance levels provided by the Central Intelligence Agency are used. These measures are derived using the 1957 production figure as the base year to which a value of 100 is assigned. Against this base value the output performance levels of all other years are measured. Thus, the performance indicator measures for other years may be higher or lower than 100 depending on their levels of output relative to that of the base year.

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APPENDIX C

Statistical Models for Hypothesis Testing

Statistical Model for Hypothesis Testing

Data analysis tests the relationship between five crisis and four policy indicators. These indicator are defined in sections 4.4.1 and 4.4.2. Consistent with hypothesis H_2 we test for causal effect of crisis on policies, as well as the reverse feedback effects. The policy indicators are examined one at a time in relation to all the crisis indicators. The reverse feedback effects are, in turn, examined. Using the autoregressive-moving average vector model our estimates of relationships for the defense expenditure indicator are:

$$
\begin{aligned}
(\text{defense expenditure/GNP})_T = & \sum_{t=T}^{T-k} \phi_{11t} \ (\text{Commander/Commissar})_t \\
& + \sum_{t=T}^{T-k} \phi_{12t} \ (\text{field army i/field army j})_t \\
& + \sum_{t=T}^{t-k} \phi_{13t} \ (\text{function i/funtion j})_t \\
& + \sum_{t-T}^{t-k} \phi_{14t} \ (\text{generation i/generation j})_t \\
& + \sum_{t=T}^{t-k} \phi_{15t} \ (\text{civilian/military})_t \\
& + N_{1t}
\end{aligned}
$$

$$(\text{commander/commissar})_T = \sum_{t=T}^{T-k} \phi_{21t} \; (\text{defense expenditure/GNP})_t + N_{2t}$$

$$(\text{field army i/field army j})_T = \sum_{t=T}^{T-k} \phi_{31t} \; (\text{defense expenditure/GNP})_t + N_{3t}$$

$$(\text{function i/funtionj})_T = \sum_{t=T}^{T-k} \phi_{41t} \; (\text{defense expenditure/GNP})_t + N_{4t}$$

$$(\text{generation i/generation j})_T = \sum_{t=T}^{T-k} \phi_{51t} \; (\text{defense expenditure/GNP})_t + N_{5t}$$

$$(\text{civilian/military})_T = \sum_{t=T}^{T-k} \phi_{61t} \; (\text{defense expenditures/GNP})_t + N_{6t}$$

where N_t is a moving average process of the form

$$N_{it} = \sum_{t=T}^{T-k} \theta_{ijt} \, a_{it} \qquad ,i=1,\ldots,6;\ j=1,\ldots,5$$

The first equation above defines the effects of the crisis indicators on a policy indicator, whereas the remaining five equations define the feedback effects of the policy on crises. These relationships are repeated for three other policy indicators: industrial/agricultural outputs, export/GNP, import/GNP.

Alternatively, we have also entertained models in which possible interdependence of the policy indicators are considered. Thus, a policy is assumed not only to be affected by crises but also by other policies. The model is a system of autoregressive-moving average vector functions. In this analysis only one crisis indicator is considered at a time in conjunction with all of the policy indicators as the following vector expression illustrates.

$$
\begin{bmatrix}
(\text{defense expenditure/GNP})_T \\
(\text{industry/agriculture})_T \\
(\text{import/GNP})_T \\
(\text{export/GNP})_T \\
(\text{commander/commissar})_T
\end{bmatrix}
=
\begin{bmatrix}
\phi_{111} & \phi_{121} \ldots & \phi_{151} \\
\phi_{211} & \phi_{221} \ldots & \phi_{251} \\
\vdots & \ddots & \vdots \\
\phi_{511} & \phi_{521} \ldots & \phi_{551}
\end{bmatrix}
\begin{bmatrix}
(\text{defense expenditure/GNP})_{T-1} \\
(\text{industry/agriculture})_{T-1} \\
(\text{import/GNP})_{T-1} \\
(\text{export/GNP})_{T-1} \\
(\text{commander/commissar})_{T-1}
\end{bmatrix}
$$

$$
+\cdots+
\begin{bmatrix}
\phi_{11k} & \phi_{12k} \ldots & \phi_{15k} \\
\phi_{21k} & \phi_{22k} \ldots & \phi_{25k} \\
\vdots & & \\
\phi_{51k} & \phi_{52k} \ldots & \phi_{55k}
\end{bmatrix}
\begin{bmatrix}
(\text{defense expenditure/GNP})_{T-k} \\
(\text{industry/agriculture})_{T-k} \\
(\text{import/GNP})_{T-k} \\
(\text{export/GNP})_{T-k} \\
(\text{commander/commissar})_{T-k}
\end{bmatrix}
$$

$$
+
\begin{bmatrix}
a_{1T} \\
a_{2T} \\
\vdots \\
a_{5T}
\end{bmatrix}
-
\begin{bmatrix}
\theta_{111} & \theta_{121} \ldots & \theta_{151} \\
\theta_{211} & \theta_{221} \ldots & \theta_{251} \\
\vdots & & \\
\theta_{511} & \theta_{521} \ldots & \theta_{551}
\end{bmatrix}
\begin{bmatrix}
a_{1T-1} \\
a_{2T-1} \\
\vdots \\
a_{5T-1}
\end{bmatrix}
$$

$$
-\cdots-
\begin{bmatrix}
\theta_{11k} & \theta_{12k} \ldots & \theta_{15k} \\
\theta_{21k} & \theta_{22k} \ldots & \theta_{25k} \\
\vdots & & \\
\theta_{51k} & \theta_{52k} \ldots & \theta_{55k}
\end{bmatrix}
\begin{bmatrix}
a_{1T-k} \\
a_{2T-k} \\
\vdots \\
a_{5T-k}
\end{bmatrix}
$$

Some results of estimations for the above models are displayed on tables 5a to 5l.

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